NUMERICAL ANALYSIS TRIAL EXAMS WEEK 14 EXERCISES

Solve the given set of exam by hand and then check the result by using computer programs, grade your exam

EXAM I

QUESTION 1

Following data is given find a least dquare curve fitting formula by using the given data

$f(x) = a_0 + a_1 x + a_2 x^2$

x	y=f(x)
0	1
0.1	1.11
0.2	1.24
0.3	1.39
0.4	1.56
0.5	1.75
0.6	1.96
0.7	2.19
0.8	2.44
0.9	2.71
1	3

QUESTION 2 $f(x) = x^3 - 6x^2 + 12x - 8$ is given. Find the root in the region xl=0.1 xu=3.0 by using **bisection method**

QUESTION 3 Solve the following integral by using Gauss-Legendre integration method. N=4

$$I = \int_{x=0}^{2} (x^3 - 6x^2 + 12x - 8)dx$$

N	x_k	c_k
4	-0.861136311594052	0.347854845137447
	-0.339981043584856	0.652145154862546
	0.339981043584856	0.652145154862546
	0.861136311594052	0.347854845137447

QUESTION 4 Solve the differential equation by using fourth order Runge-Kutta method. Initial condition x=0 y=1

$$\frac{dy}{dx} = y(x^3 - 6x^2 + 12x - 8)$$

Runge-Kutta with fourth degree polynomial solution RK4:

$y_{i+1} = y_i + (1/6)*(k_1 + 2k_2 + 2k_3 + k_4)h$

$k_1 = f(x_i, y_i)$

$k_2 = f(x_i + 0.5h, y_i + 0.5k_1 h)$

$k_3 = f(x_i + 0.5h, y_i + 0.5k_2 h)$

$k_4 = f(x_i + h, y_i + k_3 h)$

This equation can be given as Buthcher tableu as:

0	0	0	0	0
1/2	1/2	0	0	0
1/2	0	1/2	0	0
1	0	0	1	0
	1/6	2/6	2/6	1/6

QUESTION 5 Find the minimum of function by using Newton-Raphson Method

$f(x) = \frac{15x}{(4x^2 - 3x + 4)}$ the range of 0 to10.

QUESTION 6 Find the smallest eigenvalue of the given matrix

Orijinal Matrix :

12.700000000000000	-1.200000000000000	0.000000000000000
-1.200000000000000	9.700000000000000	0.000000000000000
0.000000000000000	0.000000000000000	22.500000000000000

inverse Matrix :

0.079671457905544	0.009856262833676	0.000000000000000
0.009856262833676	0.104312114989733	0.000000000000000
0.000000000000000	0.000000000000000	0.044444444444444

EXAM II

QUESTION 1

Following data is given find a least dquare curve fitting formula by using the given data

$f(x) = a_0 + a_1 x + a_2 x^2$

x	y=f(x)
0	0.5
0.1	0.675
0.2	0.9
0.3	1.175
0.4	1.5
0.5	1.875
0.6	2.3
0.7	2.775
0.8	3.3
0.9	3.875
1	4.5

QUESTION 2 $f(x) = x^3 - 6x^2 + 12x - 8$ is given. Find the root by using starting value x0=0.5 by using **Newton-Raphson method**

QUESTION 3 Solve the following integral by using Gauss-Legendre integration method. N=4

$$I = \int_{x=2}^{4} (x^3 - 6x^2 + 12x - 8)dx$$

N	x_k	c_k
4	-0.861136311594052	0.347854845137447
	-0.339981043584856	0.652145154862546
	0.339981043584856	0.652145154862546
	0.861136311594052	0.347854845137447

QUESTION 4 Solve the differential equation by using fourth order Runge-Kutta method. Initial condition x=2 y=1

$$\frac{dy}{dx} = y(x^3 - 6x^2 + 12x - 8)$$

Runge-Kutta with fourth degree polynomial solution RK4:

$y_{i+1} = y_i + (1/6)*(k_1 + 2k_2 + 2k_3 + k_4)h$

$k_1 = f(x_i, y_i)$

$k_2 = f(x_i + 0.5h, y_i + 0.5k_1h)$

$k_3 = f(x_i + 0.5h, y_i + 0.5k_2h)$

$k_4 = f(x_i + h, y_i + k_3h)$

This equation can be given as Buthcher tableu as:

0	0	0	0	0
1/2	1/2	0	0	0
1/2	0	1/2	0	0
1	0	0	1	0
	1/6	2/6	2/6	1/6

QUESTION 5 Find the minimum of function by using Golden search Method

$f(x) = \dfrac{15x}{(4x^2 - 3x + 4)}$ the range of 0 to10.

QUESTION 6 Find the smallest eigenvalue of the given matrix

Orijinal Matrix :

12.700000000000000 -1.200000000000000

-1.200000000000000 9.700000000000000

0.

inverse Matrix :

0.079671457905544 0.009856262833676

0.009856262833676 0.104312114989733

EXAM III

QUESTION 1

Following data is given find a least dquare curve fitting formula by using the given data

$f(x) = a_0 + a_1x + a_2x^2$

x	y=f(x)
0	3
0.1	3.21
0.2	3.44
0.3	3.69
0.4	3.96
0.5	4.25
0.6	4.56
0.7	4.89
0.8	5.24
0.9	5.61
1	6

QUESTION 2 $f(x) = x^3 - 6x^2 + 12x - 8$ is given. Find the root by using starting value x0=0.5 by using **Secant method**

QUESTION 3 Solve the following integral by using Gauss-Legendre integration method. N=4

$$I = \int_{x=0}^{1} \frac{\pi}{2}(1 - x^2)^{0.5} dx$$

N	x_k	c_k
4	-0.861136311594052	0.347854845137447
	-0.339981043584856	0.652145154862546
	0.339981043584856	0.652145154862546
	0.861136311594052	0.347854845137447

QUESTION 4 Solve the differential equation by using fourth order Runge-Kutta method. Initial condition x=0 y=1

$$\frac{dy}{dx} = y^2(x^3 - 6x^2 + 12x - 8)$$

Runge-Kutta with fourth degree polynomial solution RK4:

$y_{i+1} = y_i + (1/6)*(k_1 + 2k_2 + 2k_3 + k_4)h$

$k_1 = f(x_i, y_i)$

$k_2 = f(x_i + 0.5h, y_i + 0.5k_1h)$

$k_3 = f(x_i + 0.5h, y_i + 0.5k_2h)$

$k_4 = f(x_i + h, y_i + k_3h)$

This equation can be given as Buthcher tableu as:

$$\begin{array}{c|cccc} 0 & 0 & 0 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ \hline & 1/6 & 2/6 & 2/6 & 1/6 \end{array}$$

QUESTION 5 Find the minimum of function by using Secant Method

$f(x) = \dfrac{15x}{(4x^2 - 3x + 4)}$ the range of 0 to10.

QUESTION 6 Find the dominant eigenvalue of the given matrix

Orijinal Matrix :

12.700000000000000	-1.200000000000000	0.000000000000000
-1.200000000000000	9.700000000000000	0.000000000000000
0.000000000000000	0.000000000000000	22.500000000000000

inverse Matrix :

0.079671457905544	0.009856262833676	0.000000000000000
0.009856262833676	0.104312114989733	0.000000000000000
0.000000000000000	0.000000000000000	0.044444444444444