

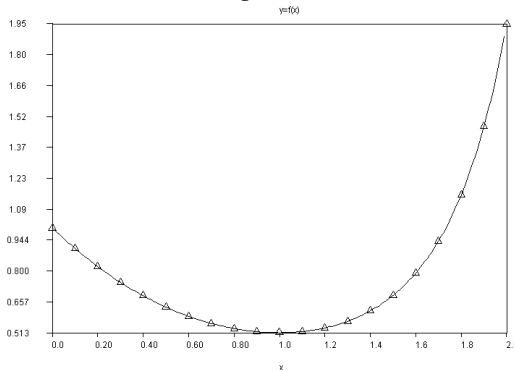
## NA2012 WEEK 12 LAB EXERCISES

### PROBLEM 1

Solve differential equation

$$\frac{dy}{dx} = yx^2 - y \quad \text{at the range } x=0 \text{ ve } x=2, \text{ with initial condition } y(0)=1.$$

- solve numerically by computer with Milne open method with  $h=0.1$
- solve numerically by computer with Hamming open method with  $h=0.1$
- solve numerically by computer with Adams-Bashford 5 step formula as predictor following Adams-Moulton 4 step formula as corrector with  $h=0.1$

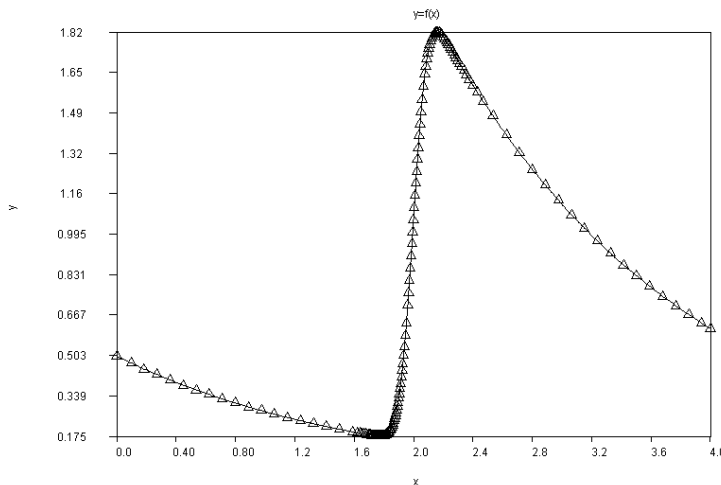


### PROBLEM 2

For differential equation

$$\frac{dy}{dx} = y * y + 10e^{-(x-2)^2 / 0.002} \quad \text{with initial condition, } y(0)=0.5 \text{ and}$$

*Variable* step size find  $y(4)$  with Runge-Kutta-Fehlberg method



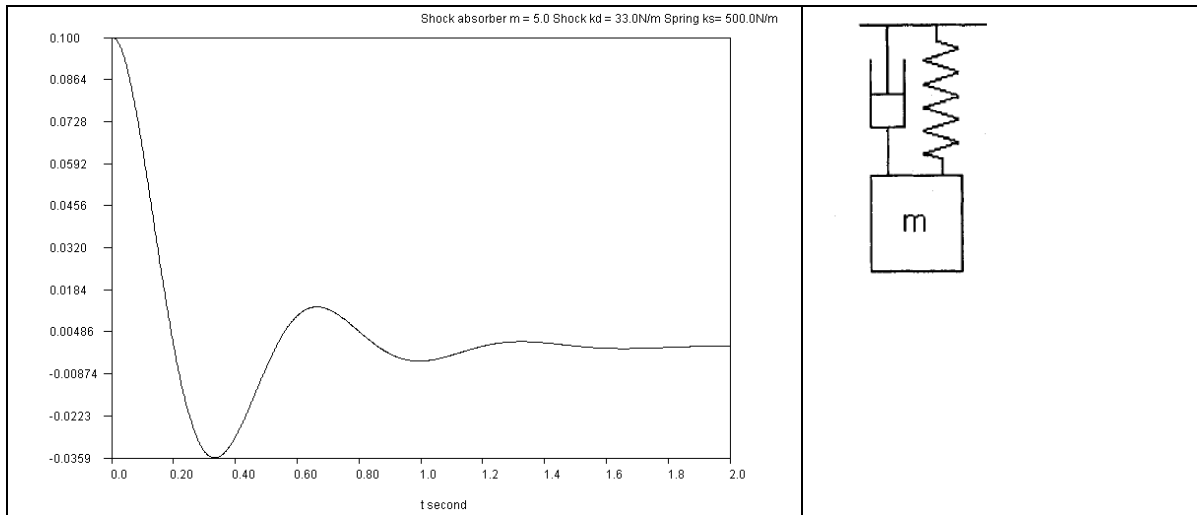
### PROBLEM 3

In cars and other machines for vibration control mechanical system with a spring and a shock absorber is used. The differential equation of such a system is given as:

$$m \frac{d^2 y}{dt^2} + k_d \frac{dy}{dt} + k_s y = 0$$

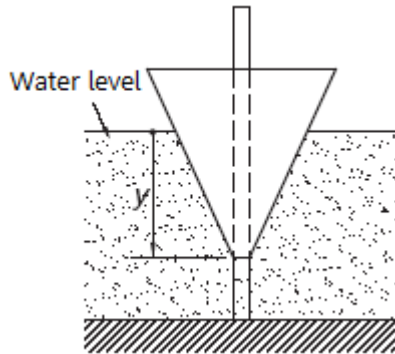
In this equation  $m$  is mass,  $k_d$  shock absorber coefficient,  $k_s$  is spring coefficient. Solve the differential

equation for  $m=5$  kg,  $k_s=500$  N/m,  $k_d=33$  N/m values. Note that  $x[0]=t$ ,  $x[1]=y$ ,  $\frac{dy}{dt} = \frac{dx[1]}{dx[0]} = x[2]$



## WEEK 11 HOMEWORKS

### PROBLEM 1



A cone buoy can slide freely on a bar is floating on the surface. If cone is disturbed by any effect, it will oscilate. The diferential equation of the oscilation is given as:

$\frac{d^2y}{dt^2} = g(1 - ay^3)$  If  $a=16 \text{ m}^{-3}$ ,  $g=9.80665 \text{ m/s}^2$  and cone position initially moved upward to  $y=0.1 \text{ m}$  and released, find the period(s) and amplitude(m) of the oscilation.

### PROBLEM 2

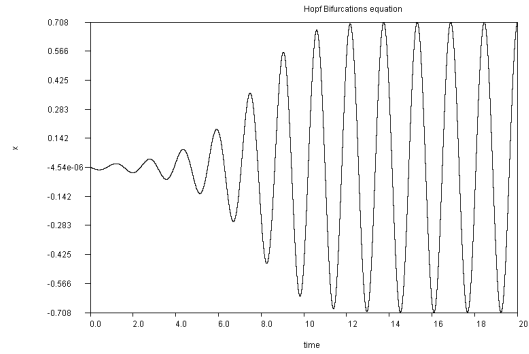
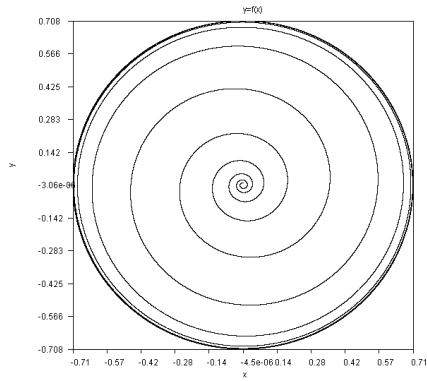
Hopf Bifurcations equation is given by

$$\frac{dx}{dt} = -(g + x^2 + y^2)x - wy$$

$$\frac{dy}{dt} = -(g + x^2 + y^2)y + wx$$

Solve the set of differential equations with  $g=-0.5, w=4$ , with the initial conditions  $x=0, y=0.01$  at  $t=0$ .

$T_{\text{final}}=20 \text{ s}$



### PROBLEM 3

Assume that a bullet is thrown from the gun into an atmosphere. Due to high velocity of the bullet, friction will effect the movement of the bullet. The differential equation of the motion in cartesian coordinate system can be given as:

$$\frac{d^2 x}{dt^2} = -\gamma(\sqrt{v_x^2 + v_y^2})v_x$$

$$\frac{d^2 y}{dt^2} = -g - \gamma(\sqrt{v_x^2 + v_y^2})v_y$$

In this equation  $\gamma = \frac{k}{m}$ , where  $k$  friction coefficient between the bullet and air,  $v_x$  is x direction component of

the velocity,  $v_y$  is y direction component of the velocity,  $m$  mass,  $g$  gravitational acceleration, and  $t$  is time.

With

$t=x[0]$  ,  $x=x[1]$ ,  $y=x[2]$ ,  $v_x=x[3]$ ,  $v_y=x[4]$

variable conversion the diferential equation becomes

$$\frac{dx[1]}{dx[0]} = x[3]$$

$$\frac{dx[2]}{dx[0]} = x[4]$$

$$\frac{dx[3]}{dx[0]} = -\gamma(\sqrt{x^2[3] + x^2[4]})x[3]$$

$$\frac{dx[4]}{dx[0]} = -g - \gamma(\sqrt{x^2[3] + x^2[4]})x[4]$$

Initial conditions  $x[0]=0$  da  $x[1]=0$   $x[2]=0$ ,

With  $V_0=500$  m/s  $x[3]= V_0 \sin(\theta)$  m/s,  $x[4]= V_0 \cos(\theta)$  m/s,  $g=9.806$  m/s<sup>2</sup>,  $\gamma=0.01$  , find initial angle  $\theta$  to hit a target at  $x=x[1]=1000$  m and  $y=x[2]=50$  m. (shooting problem)