

CUBIC PENG-ROBINSON EoS FOR DEFINING THERMODYNAMIC PROPERTIES OF HUMID AIR

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Abstract: Calculating humid air properties is the basis of psychrometry and related processes. And usually, perfect gas equation of state (EoS) is preferred for this calculation. Using perfect gas equation of state is simple and well-known method however it is not accurate for whole applications. Especially at extreme cases such as high pressures and low temperatures, error level may reach at an unacceptable level. Moreover scientists and researchers may want to use more accurate methods to calculate the properties of both humid and dry air. Therefore many different EoS have been developed for different purposes so far. But none of them can be used to obtain the whole properties of materials under all conditions.

One of these developed equations of state is Peng-Robinson EoS which was suggested by Ding-Yu Peng and Donald B. Robinson in 1976 and from this year, it has been commonly used in thermodynamic calculations. This EoS is in cubic form which enables to avoid complex iterative root finding process and this structure makes it more preferable option. Moreover, Peng-Robinson EoS defines the gas phase better than ideal gas EoS. Consequently Peng-Robinson EoS is studied in this paper for calculating thermodynamic properties like specific volume, internal energy, enthalpy, entropy and specific heat of humid air with the objective to provide a more accurate model to researchers. For this purpose program codes in java language are developed and the results are compared with ideal gas EoS. Results and comparisons are given in graphical and table form in the paper as well.

Keywords: Thermodynamic properties of humid air, Peng-Robinson EoS, Cubic EoS, IAPWS-IF97 Formulation

INTRODUCTION

Most air conditioning applications, ideal gas equation of state (EoS) is used in order to calculate thermodynamic properties of humid air. Usually it is accurate enough for ordinary processes but when applications with higher pressure zones are considered, error level will increase. Processes such as drying of humid air in a compressed air tank, adding water to the compressor of gas turbine of power plant to improve overall efficiency will require better approaches. Moreover scientists and researchers may want to use more accurate methods to calculate the properties of both humid and dry air.

Many different EoS have been developed so far. But none of them can be used to obtain the whole properties of materials under all conditions. Ideal gas EoS gives acceptable results for a wide spectrum. But it cannot be used for every case. Especially deviation from perfect gas behavior occurs most near the critical point. And at around this point, Peng-Robinson gives satisfactory

results. Moreover, it defines the gas phase better than ideal gas EoS as well (Nasri and Housam, 2009). Peng-Robinson EoS is defined as a function of critical properties and eccentric factor for pure gases and the mixing rule proposed by Harstad et al. is used to extend the Peng-Robinson equation of state to gas mixtures in this study. (Harstad et al, 1997) This EoS is in cubic form which enables to avoid complex iterative root finding process and this structure makes it more preferable option.

Since humid air consists of vapor and dry air, two different models are used to define the thermodynamic properties of each. In this study, dry air properties are defined by Peng-Robinson EoS (Peng and Robinson, 1976) and for vapor properties, IAPWS-IF97 model is preferred which was developed by The International Association for the Properties of Water and Steam in 1997 (Wagner and Kretzschmar, 2008). Definition of dry air mixture is taken from standard air formula which is given as a mixture of gases such as nitrogen, oxygen,

argon, carbon dioxide, neon, helium, methane, krypton, hydrogen and xenon (Lemmon and Javobsen, 2004).

The objective of this study is to supply to researchers a more accurate EoS for thermodynamic analysis of processes related to humid air like psychrometry. Therefore we studied Peng-Robinson EoS which was developed by Ding-Yu Peng and Donald B. Robinson in 1976 (Lopez-Echeverry et al, 2017). Set of computer programs were developed by using Peng-Robinson EoS for determining thermodynamic properties such as specific volume, internal energy, enthalpy, entropy, Gibbs energy, specific heat etc. and the result was compared with ideal gas EoS.

METHODOLOGY

Equation of State for Dry Air: Formulations of Peng Robinson

Since the dry air formulations and the solutions of equations were explained in separate study by the authors, details are not given here. Only the general formulations of Peng-Robinson EoS and the thermodynamic property equations are mentioned. Solutions and whole coefficients can be reached from article written by the authors (Çoban and Kavas, 2020).

General form of cubic equation of state is (Reid et al, 1987)

$$P = \frac{RT}{v-b} - \frac{a}{v^2+ubv+wb^2} \quad (1)$$

Peng-Robinson EoS coefficients: $u=2$, $w=-1$ so that equation took the form:

$$P = \frac{RT}{v-b} - \frac{a}{v^2+2bv-b^2} \quad (2)$$

Where

$$b = \frac{0.0780RT_{crit}}{P_{crit}} \quad (3)$$

$$a = \frac{0.45724R^2T_{crit}^2}{P_{crit}} [1 + f\omega(1 - T_r^{0.5})]^2 \quad (4)$$

$$f\omega = 0.37464 + 1.54226\omega - 0.269992\omega^2 \quad (5)$$

Parameters of the equation are defined in terms of the critical properties and the acentric factor. ω is called Pitzer's acentric factor and used in Peng-Robinson and Soave equation of states. This factor is calculated as:

$$\omega = -\log_{10} P_{saturated\ vapor}(at\ T_r = 0.7) - 1 \quad (6)$$

The reduced vapor pressure ($P_r = P/P_{crit}$) at $T_r = T/T_{crit} = 0.7$ is necessary to obtain values of ω . The equation can be written in the following form as well:

$$Z^3 - (1 + B^* - uB^*)Z^2 + (A^* + wB^{*2} - uB^* - uB^{*2})Z - A^*B^* - B^{*2} - wB^{*2} - wB^{*3} = 0 \quad (7)$$

Where

$$A^* = \frac{aP}{R^2T^2} \quad (8)$$

$$B^* = \frac{bP}{RT} \quad (9)$$

$$Z = \frac{Pv}{RT} \quad (10)$$

We are trying to establish equation of state for dry air which is a gas mixture that consists of nitrogen, oxygen, argon, carbon dioxide etc. Therefore mixing rule proposed by (Harstad et al, 1997) is used to reach dry air properties. After solving the equations and extend the solution to mixtures, thermodynamic properties can be established.

Entropy of dry air is expressed as

$$s(T, V) = s_0 + \int_{T_0}^T \frac{C_p(T)-R}{T} dT + \int_{v_0}^v \left(\frac{\partial P}{\partial T} \right)_v dv \quad (11)$$

Internal energy of dry air can be derived similarly as

$$u(T, v) = u_0 + \int_{T_0}^T (C_p(T) - R) dT + \int_{v_0}^v \left(T \left(\frac{\partial P(T, v)}{\partial T} \right)_v - P(T, v) \right) dv \quad (12)$$

Enthalpy of dry air:

$$h(T, v) = u + vP(T, v) \quad (13)$$

Specific heat at constant pressure is expressed as:

$$C_p(T) = \sum_{i=0}^{n-1} y_i C_{pi}(T) = \sum_{i=0}^{n-1} y_i \left[A_i + B_i 10^{-3} T + \frac{C_i 10^5}{T^2} + D_i 10^{-6} T^2 \right] \quad (14)$$

Specific heat at constant volume is expressed as:

$$C_v(T) = C_p(T) - R = \sum_{i=0}^{n-1} y_i C_{vi}(T) \quad (15)$$

Equation of State for Vapor: Formulations of IAPWS-IF97

In order to get the properties of vapor in humid air, IAPWS-IF97 is used. This formulation was developed by International Association for the Properties of Water and Steam(IAPWS) in 1997. The IAPWS IF97 formulation covers the following range of validity (Wagner and Kretzschmar, 2008):

$$273.15\text{ K} \leq T \leq 1073.15\text{ K}, \quad 0 < p \leq 100\text{ MPa}$$

$$1073.15\text{ K} < T \leq 2273.15\text{ K}, \quad 0 < p \leq 50\text{ MPa}$$

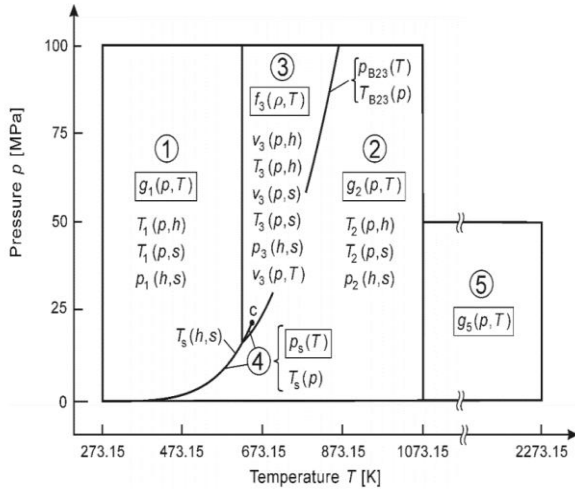


Figure 1. Equation of state regions in IAPWS-IF97 (Wagner and Kretzschmar, 2008)

The IAPWS IF97 formulation is divided in 5 regions as shown in Figure 1. The first equation, which covers basically liquid region, has the following Gibbs free energy form:

$$\frac{g_1(p, T)}{RT} = \gamma(\pi, \tau) = \sum_{i=1}^{34} n_i (71 - \pi)^{J_i} (\tau - 1222)^{J_i} \quad (16)$$

Where $\pi = p / p^*$, $\tau = T^* / T$, $p^* = 16.53$ MPa, $T^* = 1386$ K and R which represents specific gas constant of ordinary water has the value of 0.461526 kJ/(kgK). In all 5 regions, specific gas constant of ordinary water is used for calculations. Coefficients of Equation 16 are given in Table 1 as well.

Table 1. Coefficients of equation 16 (IAPWS-IF97).

i	I _i	J _i	n _i	i	I _i	J _i	n _i
1	0	-2	0.146329	18	2	3	-4.414184E-06
2	0	-1	-0.845481	19	2	17	-7.269499E-16
3	0	0	-3.756360	20	3	-4	-3.167964E-05
4	0	1	3.385516	21	3	0	-2.827079E-06
5	0	2	-0.957919	22	3	6	-8.520512E-10
6	0	3	0.157720	23	4	-5	-2.242528E-06
7	0	4	-0.016616	24	4	-2	-6.517122E-07
8	0	5	0.000812	25	4	10	-1.434172E-13
9	1	-9	0.000283	26	5	-8	-4.051699E-07
10	1	-7	-0.000607	27	8	-11	-1.273430E-09
11	1	-1	-0.018990	28	8	-6	-1.742481E-10
12	1	0	-0.032529	29	21	-29	-6.876213E-19
13	1	1	-0.021841	30	23	-31	1.4478307E-20
14	1	3	-0.000052	31	29	-38	2.6335781E-23
15	2	-3	-0.000471	32	30	-39	-1.194762E-23
16	2	0	-0.000300	33	31	-40	1.822809E-24
17	2	1	0.000047	34	32	-41	-9.353708E-26

Thermodynamic properties can be calculated from these thermodynamic relations.

Specific volume:

$$v = \left(\frac{\partial g}{\partial p} \right)_T \quad (17)$$

Specific enthalpy:

$$h = g - T \left(\frac{\partial g}{\partial p} \right)_p \quad (18)$$

Specific internal energy:

$$u = g - T \left(\frac{\partial g}{\partial T} \right)_p - p \left(\frac{\partial g}{\partial p} \right)_T \quad (19)$$

Specific entropy:

$$s = \left(\frac{\partial g}{\partial T} \right)_p \quad (20)$$

Specific isobaric heat capacity:

$$C_p = \left(\frac{\partial h}{\partial T} \right)_p \quad (21)$$

Specific isochoric heat capacity:

$$C_v = \left(\frac{\partial u}{\partial T} \right)_v \quad (22)$$

Equation 23 identified the second region, which covers vapor region has the following Gibbs free energy form:

$$\frac{g_2(p, T)}{RT} = \gamma(\pi, \tau) = \gamma^0(\pi, \tau) + \gamma^r(\pi, \tau) \quad (23)$$

In Equation 23, term $\gamma^0(\pi, \tau)$ represents ideal gas and $\gamma^r(\pi, \tau)$ is the real gas part of the EoS. For the ideal gas part, the term can be expressed as:

$$\gamma^0(\pi, \tau) = \ln \pi + \sum_{i=1}^9 n_i^0 \tau^{J_i} \quad (24)$$

Where $p^* = 1$ MPa and $T^* = 540$ K and the related coefficients are given in Table 2.

Table 2. Coefficients of equation 24 (IAPWS-IF97).

i	J _i	ni0	i	J _i	ni0
1	0	-9.692768E+00	6	-2	1.424081E+00
2	1	1.0086655E+01	7	-1	-4.383951E+00
3	-5	-5.608791E-03	8	2	-2.840863E-01
4	-4	7.145273E-02	9	3	2.1268463E-02
5	-3	-4.07104E-01			

Dimensionless residual part of the Equation 23 is as follows:

$$\gamma^r(\pi, \tau) = \sum_{i=1}^{43} n_i \pi^{I_i} (\tau - 0.5)^{J_i} \quad (25)$$

Where $p^* = 1$ MPa and $T^* = 540$ K

Table 3. Coefficients of equation 25 (IAPWS-IF97).

i	li	Ji	ni	i	li	Ji	ni
1	1	0		7	0		-5.90595E-18
2	1	1	-1.783486E-02	24	7	11	-1.26218E-06
3	1	2	-4.599601E-02	25	7	25	-3.89468E-02
4	1	3	-5.758125E-02	26	8	8	1.125621E-11
5	1	6	-5.032527E-02	27	8	36	-8.23113E+00
6	2	1	-3.303264E-05	28	9	13	1.980971E-08
7	2	2	-1.894898E-04	29	10	4	1.040696E-19
8	2	4	-3.939277E-03	30	10	10	-1.02347E-13
9	2	7	-4.379729E-02	31	10	14	-1.00181E-09
10	2	36	-2.667454E-05	32	16	29	-8.08829E-11
11	3	0	2.048173E-08	33	16	50	1.069303E-01
12	3	1	4.387066E-07	34	18	57	-3.36622E-01
13	3	3	-3.227767E-05	35	20	20	8.918584E-25
14	3	6	-1.503392E-03	36	20	35	3.062931E-13
15	3	35	-4.066825E-02	37	20	48	-4.20024E-06
16	4	1	-7.884730E-10	38	21	21	-5.90560E-26
17	4	2	1.279071E-08	39	22	53	3.782694E-06
18	4	3	4.822537E-07	40	23	39	-1.27686E-15
19	5	7	2.292207E-06	41	24	26	7.308761E-29
20	6	3	-1.671476E-11	42	24	40	5.541471E-17
21	6	16	-2.117147E-03	43	24	58	-9.43697E-07
22	6	35	-2.389574E+01				

Region 3 equation is given as Helmholtz free energy form:

$$\frac{f_3(\rho, T)}{RT} = \phi(\delta, \tau) = n_1 \ln \delta + \sum_{i=2}^{40} n_i \delta^{l_i} \tau^{J_i} \quad (26)$$

Where, $\delta = \rho / \rho^*$ $\tau = T^* / T$, $T^* = T_c = 647.096\text{K}$ and $R = 0.461526 \text{ kJ}/(\text{kgK})$ and the coefficients of Equation are given in Table 4.

Table 4. Coefficients of equation 26 (IAPWS-IF97).

i	li	Ji	ni	i	li	Ji	ni
1	0	0	1.06580E+00	21	3	4	-2.01899E+00
2	0	0	-1.57328E+01	22	3	16	-8.2147E-03
3	0	1	2.09443E+01	23	3	26	-4.7596E-01
4	0	2	-7.68677E+00	24	4	0	4.398407E-02
5	0	7	2.61859E+00	25	4	2	-4.4476E-01
6	0	10	-2.80807E+00	26	4	4	9.05720E-01
7	0	12	1.20533E+00	27	4	26	7.05224E-01
8	0	23	-8.45668E-03	28	5	1	1.07705E-01
9	1	2	-1.26543E+00	29	5	3	-3.2913E-01
10	1	6	-1.15244E+00	30	5	26	-5.0871E-01
11	1	15	8.852104E-01	31	6	0	-2.2175E-02
12	1	17	-6.42077E-01	32	6	2	9.42607E-02
13	2	0	3.84934E-01	33	6	26	1.64362E-01
14	2	2	-8.52147E-01	34	7	2	-1.3503E-02
15	2	6	4.89722E+00	35	8	26	-1.4834E-02
16	2	7	-3.05026E+00	36	9	2	5.79229E-04
17	2	22	3.942053E-02	37	9	26	3.23089E-03
18	2	26	1.255840E-01	38	10	0	8.09648E-05
19	3	0	-2.7999E-01	39	10	1	-1.65576E-04
20	3	2	1.38997E+00	40	11	26	-4.49238E-05

It should be noted that this set of equation is function of density and temperature and basic equation is Helmholtz equation so, let us list definition of other thermodynamic properties.

Pressure:

$$p = \rho^2 \left(\frac{\partial f}{\partial \rho} \right)_T \quad (27)$$

Specific enthalpy:

$$h = f - T \left(\frac{\partial f}{\partial T} \right)_p + \rho \left(\frac{\partial f}{\partial \rho} \right)_T \quad (28)$$

Specific internal energy:

$$u = f - T \left(\frac{\partial f}{\partial T} \right)_p \quad (29)$$

Specific entropy:

$$s = \left(\frac{\partial f}{\partial T} \right)_p \quad (30)$$

Specific isobaric heat capacity:

$$C_p = \left(\frac{\partial h}{\partial T} \right)_p \quad (31)$$

Specific isochoric heat capacity:

$$C_v = \left(\frac{\partial u}{\partial T} \right)_v \quad (32)$$

Region 4 of the IAPWS IF-97 equation defines saturation region. The basic equation is given as a polynomial:

$$\beta^2 \vartheta^2 + n_1 \beta^2 \vartheta + n_2 \beta^2 + n_3 \beta \vartheta^2 + n_4 \beta \vartheta + n_5 \beta + n_6 \vartheta^2 + n_7 \vartheta + n_8 = 0 \quad (33)$$

Where

$$\beta = (p_s / p^*)^{0.25} \quad (34)$$

$$\vartheta = \frac{T_s}{T^*} + \frac{n_9}{\left(\frac{T_s}{T^*} \right) - n_{10}} \quad (35)$$

From this equation both saturation pressure and saturation temperature equation can be derived.

$$\frac{P_s}{P^*} = \left[\frac{2C}{-B + (B^2 - 4AC)^{0.5}} \right]^4 \quad (36)$$

Where $p^* = 1 \text{ MPa}$

$$A = \vartheta^2 + n_1 \vartheta + n_2 \quad (37)$$

$$B = n_3 \vartheta^2 + n_4 \vartheta + n_5 \quad (38)$$

$$C = n_6 \vartheta^2 + n_7 \vartheta + n_8 \quad (39)$$

Table 5. Coefficients of equation 37-38-39 (IAPWS-IF97).

i	n _i	i	n _i
1	1.167052E+03	6	1.4915108E+01
2	-7.24213E+05	7	-4.823265E+03
3	-1.707384E+01	8	4.0511340E+05
4	1.2020824E+04	9	-2.385555E-01
5	-3.2325550E+06	10	6.5017534E+02

It is also possible to drive saturation temperature equation from the basic polynomial as:

$$\frac{T_s}{T^*} = \frac{n_{10} + D - \left[(n_{10} + D)^2 - 4(n_9 + n_{10}D) \right]^{0.5}}{2} \quad (40)$$

Where $T^*=1$ K

$$D = \frac{2G}{-F - (F^2 - 4EG)^{0.5}} \quad (41)$$

$$E = \beta^2 + n_3\beta + n_6 \quad (42)$$

$$F = n_1\beta^2 + n_4\beta + n_7 \quad (43)$$

$$G = n_2\beta^2 + n_5\beta + n_8 \quad (44)$$

And the final region which covers high temperature zone for vapor is region 5, again given as gibbs free equation type of EoS.

$$\frac{g_5(p, T)}{RT} = \gamma(\pi, \tau) = \gamma^0(\pi, \tau) + \gamma^r(\pi, \tau) \quad (45)$$

Where $\pi = p/p^*$ $\tau = T^*/T$ $R=0.461526$ kJ/(kgK),

Like in region 5, $\gamma^0(\pi, \tau)$ is the ideal gas part of EoS and $\gamma^r(\pi, \tau)$ is the real gas part of the EoS. For the ideal gas part, the term can be expressed as:

$$\gamma^0(\pi, \tau) = \ln \pi + \sum_{i=1}^9 n_i^0 \tau^{J_i} \quad (46)$$

Where $p^*=1$ MPa and $T^*=1000$ K

Table 6. Coefficients of equation 46 (IAPWS-IF97).

i	J _i ⁰	n _i	i	J _i ⁰	n _i
1	0	-13.17998	4	-2	0.3690158
2	1	6.8540841	5	-1	-3.116131
3	-3	-0.024805	6	2	-0.329616

The real gas part of the Equation 45 is defined as:

$$\gamma^r(\pi, \tau) = \sum_{i=1}^{43} n_i \pi^{I_i} \tau^{J_i} \quad (47)$$

Table 7. Coefficients of equation 47 (IAPWS-IF97).

i	I _i	J _i	n _i	i	I _i	J _i	n _i
1	1	1	1.5736404E-03	4	2	3	2.24400E-06
2	1	2	9.0153761E-04	5	2	9	-4.11632E-06
3	1	3	-5.0270077E-03	6	3	7	3.79194E-08

At reference (Çoban, 2019) more detailed set of equations are given for this calculations, for example reverse equation sets are given so that instead of additional curve fitting to get the variables, this additional sets can be utilized.

Formulations for Humid Air

After establishing dry air and moisture models, equations for humid air can be generated. Thermodynamic properties are expressed according to mass fractions of component in humid air mixture. Mass fraction of dry air, enthalpy and entropy terms are given in Equations 48, 49 and 50 respectively.

$$A = \frac{x_A}{1 - (1 - x_A)[1 - (M_W/M_A)]} \quad (48)$$

$$h^{AV}(A, T, \rho) = (1 - A)h^V(T, \rho^V) + Ah^A(T, \rho^A) \quad (49)$$

$$s^{AV}(A, T, \rho) = (1 - A)s^V(T, \rho^V) + As^A(T, \rho^A) \quad (50)$$

RESULTS

In order to get numerical values and graphical outputs, computers codes were developed in java language. The list of these codes is given in Table 8. As it can be seen from the table, humid_ir_PG is perfect gas EoS and humid_air_PR is Peng Robinson EoS for humid air.

Results obtained from Peng Robinson EoS are compared with ideal gas EoS and the differences between them are shown in graphics. Graphics were created according to temperature and pressure variation for different specific humidity.

Table 8. List of programs developed for calculating thermodynamic properties

Group/State	Model	Reference and Description
Dry Air	air_PR	Peng Robinson EoS for dry air [Reid et al, 1987)]
Dry Air	air_PG	Perfect gas EoS as a single gas for dry air.
Humid Air	humid_air_PG	Humid air EoS, using perfect gas EoS
Humid Air	humid_air_PR	Humid air EoS using Peng Robinson EoS for dry air, and IAPWS IF97 for moisture

Graphic output of Peng Robinson EoS dry air program is given in Figure 2 as an example. As given in the output,

all thermodynamic and thermophysical properties are calculated for given conditions.

	0	1	2	3
0		P, pressure	100.0	kPa
1		T, temperature	300.0	deg K
2		v, specific volume	0.86068925616902...	m ³ /kg
3		h, enthalpy	27.4978677275264...	KJ/kg
4		u, internal energy	19.7667000505451...	KJ/kg
5		s, entropy	0.09428990072652...	KJ/kgK
6		x, quality	2.0	kg vap/kg mix
7		Cp, specific heat at...	1.00484279166310...	KJ/kgK
8		Cv, specific heat at...	0.71779290736475...	KJ/kgK
9		β isobaric thermal ...	0.00334399246971...	1/K
10		κ isothermal comp...	9.97509391205509...	kPa
11		η Dynamic viscosity	1.85581523427694...	Pa.s
12		κ Thermal conduct...	0.02623176260023...	W/mK
13		σ Surface tension	0.0	N/m
14		Prandtl number	0.71234042678339...	
15		ρ density	1.16185951298039...	kg/m ³
16		speed of sound	347.5475043590632	m/s

Figure 2. Peng Robinson EoS for dry air program graphic output for P=100 kPa and T=300 K.

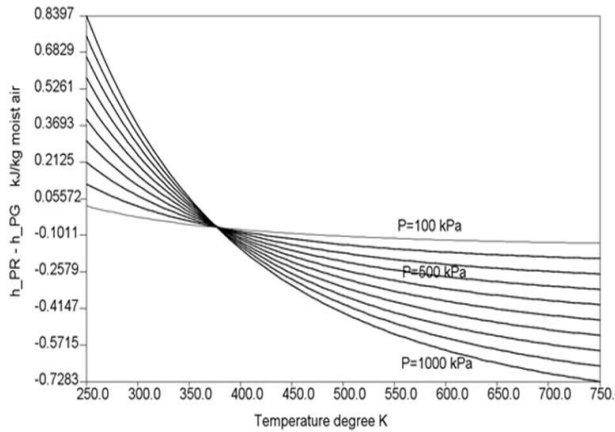


Figure 3. Enthalpy difference between Peng-Robinson and ideal gas EoS at w=0.001 kg moisture/kg dry air

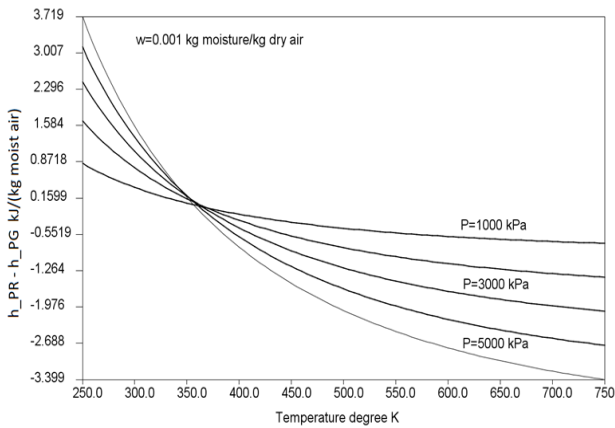


Figure 4. Enthalpy difference between Peng-Robinson and ideal gas EoS at w=0.001 kg moisture/kg dry air (for above 1000 kPa).

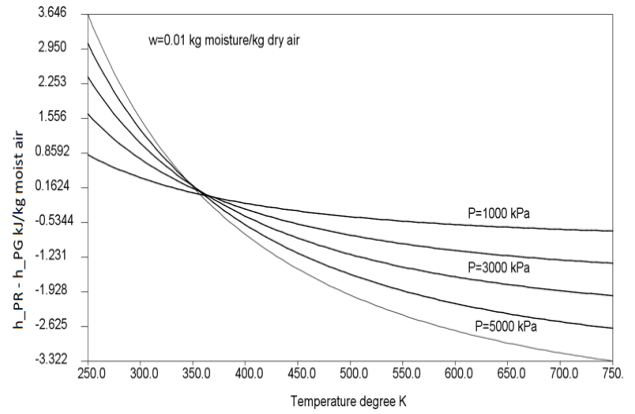


Figure 5. Enthalpy difference between Peng-Robinson and ideal gas EoS at w=0.01 kg moisture/kg dry air (for above 1000 kPa).

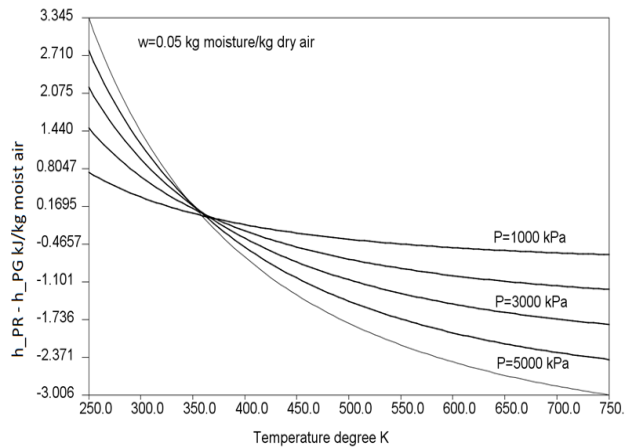


Figure 6. Enthalpy difference between Peng-Robinson and ideal gas EoS at w=0.05 kg moisture/kg dry air (for above 1000 kPa).

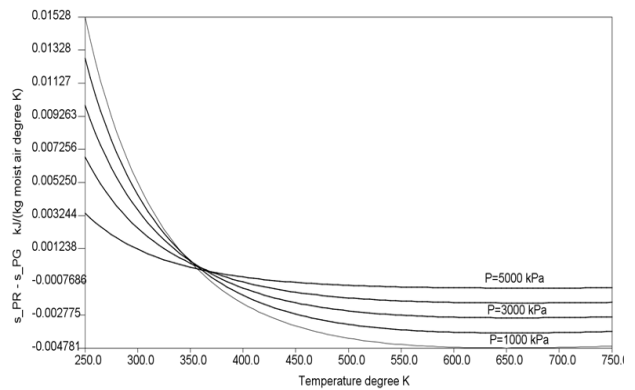


Figure 7. Entropy difference between Peng-Robinson and ideal gas EoS at w=0.001 kg moisture/kg dry air (for above 1000 kPa).

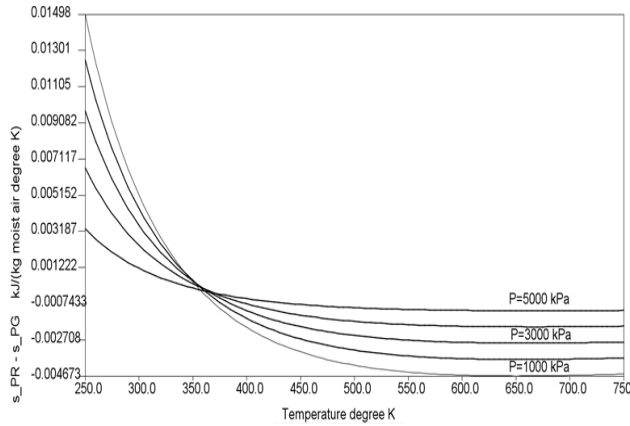


Figure 8. Entropy difference between Peng-Robinson and ideal gas EoS at $w=0.01$ kg moisture/kg dry air (for above 1000 kPa).

CONCLUSION

Results obtained for several thermodynamic states by using different EoS compared in Table 9.

As it can be both seen from graphics and Table 9, ideal gas and Peng-Robinson EoS give results close to each other for low pressures. Temperature change does not have much effect at low pressures. But at high pressures where gases moving away from the ideal gas behavior, divergence increases notably. The relative error of enthalpy between two EoS is 3.206% at 5000 kPa, and 298.15K.

The absolute humidity affects the divergence very slightly which can be observed from Figure 4,5,6 and can be assumed negligible.

Consequently Peng-Robinson EoS gives realistic results at high pressure zones. In other cases, there is not much difference between them. Using Peng-Robinson EoS is not required for low pressure values. As it can be seen from Figure 3, the enthalpy difference is in acceptable ranges below 1000 kPa and may be used for psychrometric processes. But error level is not suitable for scientific studies. Ideal gas EoS can be an option for industrial utilization only but with the help of computation power of today, realistic models should be preferred and obtained accurate results in any cases.

All equations for properties of humid air are given with details and coefficients in the article for the researchers who wish to develop their own version of computer codes. Similar equations for dry air can be also got from Çoban and Kavas, 2020.

Whole set of computer codes developed in java programming language for researchers and graphic user interface programs for non-researchers are given as free access to utilize in their own studies at internet address www.turhancoban.com as well.

Table 9. Comparison of results for $w=0.01$ kg moisture/kg dry air

Humid Air Model	Pressure (kPa)	Temperature (K)	Enthalpy (kJ/kg)	Absolute Error	Relative Error	Entropy (kJ/kgK)	Absolute Error	Relative Error
Humid Air PR	101.325	298.15	50,141486	-0,021540	0,043%	0,179708	-0,000097	0,054%
Humid Air PG	101.325	298.15	50,119946			0,179611		
Humid Air PR	5000	298.15	48,505665	1,606663	3,206%	-0,937009	0,005401	0,580%
Humid Air PG	5000	298.15	50,112328			-0,931608		
Humid Air PR	101.325	373.15	126,308970	-0,065403	0,052%	0,407630	-0,000246	0,060%
Humid Air PG	101.325	373.15	126,243567			0,407384		
Humid Air PR	5000	373.15	126,448085	-0,302543	0,240%	-0,717833	-0,000892	0,124%
Humid Air PG	5000	373.15	126,145542			-0,718725		

NOMENCLATURE

A	Mass fraction of dry air in humid air (kg/kg)	f	Specific Helmholtz free energy
a	Cubic equation coefficient (Pa kmol/m^3)	F_w	A function of w specific for Peng-Robinson EoS
b	Cubic equation coefficient (m^3/kmol)	g	Specific Gibbs energy of humid air (J/kg)
C_p	Specific heat capacity at constant pressure (J/kg K)	h	Enthalpy (kJ/kg)
C_{pi}	Individual gas specific heat capacity at constant pressure (J/kg K)	M	Molecular weight (kg/kmol)
C_v	Specific heat capacity at constant volume (J/kg K)	P	Pressure (kPa)
C_{vi}	Individual gas specific heat capacity at constant volume (J/kg K)	R	Universal molar gas constant (8.314 J/mol K)
		s	Entropy (kJ/kg K)
		T	Temperature (K)

T*	132,5 K
u	Internal energy (kJ/kg)
v	Specific volume (m ³ /kg)
w	Absolute humidity(kg moisture/kg dry air)
V	Volume (m ³)
Z	Compressibility factor

Greek symbols

ω	Pitzer's acentric factor
ρ	Density (kg/m ³)
ρ^*	314.3 (kg/m ³)
β	Transformed pressure
γ	Dimensionless Gibbs free energy, ($\gamma = g / RT$)
δ	Reduced density, ($\delta = \rho / \rho^*$)
ϑ	Transformed temperature
π	Reduced pressure, ($\pi = p / p^*$)
τ	Inverse reduced temperature, ($\tau = T^* / T$)
ϕ	Dimensionless Helmholtz free energy, ($\phi = f / RT$)
υ	Molar volume (m ³ /kmol)

Superscripts:

O	Ideal-gas part
r	Residual part
*	Reducing quantity
'	Saturated liquid state
"	Saturated vapor state

Subscripts

crit	critical value
r	reduced value

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