

# THERMODYNAMIC AND THERMOPHYSICAL PROPERTIES OF HUMID AIR BY USING CUBIC PENG-ROBINSON EOS

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## 1. ABSTRACT

In humid air thermodynamic property calculations, it is usually ideal gas EOS is used. The basic reason for this is simplicity of using ideal gas EOS. For most air condition applications it might be sufficient, but when applications with higher pressure zones are considered error level will increased. An equation of state with better accuracy of thermodynamic properties will be required for extreme cases. In this study Peng-Robinson equation of state will be used to establish computer programs in java language to calculate properties of humid air. Cubic equation of states are basically defining the gas phase, with a better accuracy compare to ideal gas EOS and can be defined accurate enough results for high pressure applications as well. It is also possible to solve inverse equation  $v(T,P)$  by using cubic form of the equation. Avoiding of more complex iterative root finding process, makes cubic equations a desirable EOS selection. For pure gases Peng-Robinson EOS defined as a function of critical properties and acentric factor. In order to mix gases Harstad, Miller, and Bellan EOSs for gas mixtures, particularly of PR-EOS. In this study the mixing rule proposed by Miller et al. is used to extend the PR equation of state to mixtures. Definition of dry air mixture is taken from standard air formula which is given as a mixture of gases such as Nitrogen, Oxygen, Argon, carbon dioxide, Neon, Helium, Methane, Krypton, Hydrogen and Xeonon . Mixing of air and water is carried out by using PR EOS mixing, but water properties such as saturation vapor pressure, saturation liquid and vapor densities are taken by utilizing IAPWS-97 water and steam industrial EOS. Basic thermodynamic and heat-mass transfer equations are used to define properties such as dew point temperature, adiabatic saturation temperature and wet bulb temperature. In order to calculate thermophysical properties such as viscosity, thermal conductivity etc. combination of cubic surface splines and Wilke, Reichenberg... methods are used. Results of developed equations are compared with Ideal gas and IAPWS-95 EOS & IAPWS G8-10 Humid air EOS. A Graphical user interface is also developed. All the program codes are listed as free access in [www.turhancoban.com](http://www.turhancoban.com) address. As subprograms properties of properties of dry air can also be calculated by using this set of PR EOS programs. All the codes are developed in java programming language.

Key Words: Thermodynamic properties of humid air, Peng-Robinson EOS, cubic EOS, thermodynamic properties of dry air, thermophysical properties of humid air

## 2. FORMULATIONS OF THE EQUATION OF STATES

We will consider Peng-Robinson cubic equation of states for dry exhaust gas mixture in this paper. Details of the Peng-Robinson Equation of State is given below.

Cubic Equation of State has a general form of equation

$$P = \frac{RT}{v-b} - \frac{a}{v^2+ubv+wb^2} \quad 2.1$$

Peng-Robinson EOS coefficients:  $u=2, w=-1$  so that equation took the form:

$$P = \frac{RT}{v-b} - \frac{a}{v^2+2bv-b^2} \quad 2.2 \text{ where}$$

$$b = \frac{0.0780RT_{crit}}{P_{crit}} \quad 2.3 \text{ and}$$

$$a = \frac{0.45724R^2T_{crit}^2}{P_{crit}} [1 + f\omega(1 - T_r^{0.5})]^2 \quad 2.4$$

$$f\omega = 0.37464 + 1.54226\omega - 0.269992\omega^2 \quad 2.5$$

$\omega$  in Peng-Robinson and Soawe equation of states coefficient is called **acentric factor**. This factor is calculated as

$$\omega = -\log_{10} P_{saturated \ vapor} (\text{at } T_r = 0.7) - 1 \quad 2.6$$

To obtain values of  $\omega$ , the reduced vapor pressure ( $P_r = P/P_{crit}$ ) at  $T_r = T/T_{crit} = 0.7$  is required.

The equation can also be written in the following form:

$$Z^3 - (1 + B^* - uB^*)Z^2 + (A^* + wB^{*2} - uB^* - uB^{*2})Z - A^*B^* - B^{*2} - wB^{*2} - wB^{*3} = 0 \quad 2.7$$

$$\text{where } A^* = \frac{aP}{R^2T^2} \quad 2.8 \quad \text{and } B^* = \frac{bP}{RT} \quad 2.9 \quad Z = \frac{Pv}{RT} \quad 2.10$$

We are trying to establish equation of state which is a gas mixture.

More recently, Harstad, Miller, and Bellan [1] have presented computationally efficient forms of EOSs for gas mixtures, particularly of PR-EOS. They have also shown that it is possible to extend the equations' validity beyond the range of data using departure functions.

In this study the mixing rule proposed by Miller et al. is used to extend the PR equation of state to mixtures. In particular, the parameters  $a$  and  $b$  can be obtained by

$$a = \sum_i \sum_j y_i y_j a_{ij} \quad 2.11 \text{ and } b = \sum_i y_i b_i \quad 2.12 \text{ where } y \text{ is the mole fraction in the vapor phase}$$

$$\text{Where } b_{ij} = \frac{0.0780RT_{crit \ ij}}{P_{crit \ ij}} \quad 2.13 \quad a_{ij} = \frac{0.45724R^2T_{crit \ ij}^2}{P_{crit \ ij}} [1 + f\omega_{ij}(1 - T_{r \ ij}^{0.5})]^2 \quad 2.14$$

$$f\omega_{ij} = 0.37464 + 1.54226\omega_{ij} - 0.269992\omega_{ij}^2 \quad 2.15$$

$$T_{r \ ij} = T/T_{crit \ ij} \quad 2.16$$

The diagonal elements of the “critical” matrices are equal to their corresponding pure substance counterparts, i.e.,  $T_{crit \ ii} = T_{crit \ i}$ ,  $P_{crit \ ii} = P_{crit \ i}$ , and  $\omega_{ii} = \omega_i$ . The off-diagonal elements are evaluated through additional rules:

$$P_{crit \ ij} = \frac{Z_{crit \ ij} RT_{crit \ ij}}{V_{crit \ ij}} \quad 2.17$$

$$V_{crit \ ij} = \frac{1}{8} [(V_{crit \ ii})^{1/3} + (V_{crit \ jj})^{1/3}]^3 \quad 2.18$$

$$Z_{crit \ ij} = \frac{1}{2} [Z_{crit \ ii} + Z_{crit \ jj}] \quad 2.19$$

$$\omega_{ij} = \frac{1}{2} [\omega_{ii} + \omega_{jj}] \quad 2.20$$

$$T_{crit \ ij} = \sqrt{T_{crit \ ii} T_{crit \ jj}} (1 - k_{ij}) \quad 2.21$$

Where interaction coefficient  $k_{ij}$  can be calculated as:

$$k_{ij} = 1 - \frac{(V_{crit \ ii} V_{crit \ jj})^{1/2}}{V_{crit \ ij}} \quad 2.22$$

Partial derivatives with respect to a

$$\frac{\partial a}{\partial T} = -\frac{1}{T} \sum_i \sum_j \left( y_i y_j a_{ij} \frac{f \omega_{ij} \sqrt{T_{rij}}}{1 + f \omega_{ij} (1 - \sqrt{T_{rij}})} \right) \quad 2.23$$

$$\frac{\partial^2 a}{\partial T^2} = \frac{0.457236 R^2}{2T} \sum_i \sum_j \left( y_i y_j a_{ij} (1 - f \omega_{ij}) \frac{T_{crit,ij}}{P_{crit,ij}} \sqrt{T_{rij}} \right) \quad 2.24$$

In order to solve v(T,P) root solving is required, but due to structure of cubic equations root solving can easily be established by using Tartaglia & Cardino formula(1530). The basic formulas used to calculate cubic roots analytically are as follows:

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 \quad 2.25$$

$$a = a_2/a_3 \quad b = a_1/a_3 \quad c = a_0/a_3 \quad 2.26$$

$$y = c + bx + ax^2 + x^3 \quad 2.27$$

$$Q = \frac{a^2 - 3b}{9} \quad z = 2a^3 - 9ab + 27c \quad 2.28 \quad R = z/54 \quad 2.29$$

if ( $R^2 < Q^3$ )

$$\left\{ \begin{array}{l} \theta = \cos^{-1} \left( \frac{R}{\sqrt[3]{Q^3}} \right) \\ x_0 = -2\sqrt{Q} \cos[\theta/3] - a/3 \\ x_1 = 2\sqrt{Q} \cos[(\theta - 2\pi)/3] - a/3 \\ x_2 = 2\sqrt{Q} \cos[(\theta - 2\pi)/3] - a/3 \end{array} \right.$$

else

$$\left\{ \begin{array}{l} A = -(R + \sqrt{R^2 - Q^3})^{1/3} \\ \text{if}(a == 0) B = 0 \\ \text{else } B = Q/A \\ x_0 = (A + B - a/3) \\ x_1 = \left[ \left( -\frac{A+B}{2} \right) - a/3 \right] + \left[ \frac{\sqrt{3(A-B)}}{2} \right] i \\ x_2 = \left[ \left( -\frac{A+B}{2} \right) - a/3 \right] - \left[ \frac{\sqrt{3(A-B)}}{2} \right] i \end{array} \right. \quad 2.30$$

Data is also needed to solve  $C_p(T)$  value. In order to establish that, NIST tables given at the address <https://janaf.nist.gov/> is used. The following partial continuous formulation is taken.

Since  $C_p(T)$  value is for the ideal gas, ideal gas mixing rule applied to establish  $C_p(T)$  value of the mixture from the given gases. For each individual gases the following partial difference curve fitting formula is used

$$C_{pi}(T) = A_i + B_i 10^{-3} T + \frac{C_i 10^5}{T^2} + D_i 10^{-6} T^2 \quad T_{Li} \leq T \leq T_{Hi} \quad 2.31$$

Table 2.1 Composition and critical properties of dry exhaust gas

Name	Formula	Molar mass	Tc deg K	Pc bar	Zc	$\omega$
Nitrogen	N <sub>2</sub>	28.013	126.2	33.9	0.29	0.039
Oxygen	O <sub>2</sub>	31.999	154.6	50.4	0.288	0.025
Argon	Ar	39.948	150.8	48.7	0.291	0.001
Carbondioxide	CO <sub>2</sub>	44.01	304.1	73.8	0.274	0.239
Carbonmonoxide	CO	28.01	132.86	34.94	0.2915148	0.0497
Hydrogen	H <sub>2</sub>	2.016	33	12.9	0.303	-0.216
Methane	CH <sub>4</sub>	16.42	190.4	46	0.288	0.011
Sulphuredioxide	SO <sub>2</sub>	64.063	430.8	78.8	0.269	0.256
Nitrogenoxide	NO	30.006	180	64.8	0.25	0.588
Nitrogendioxide	NO <sub>2</sub>	46.006	261.9	101	0.473	0.834

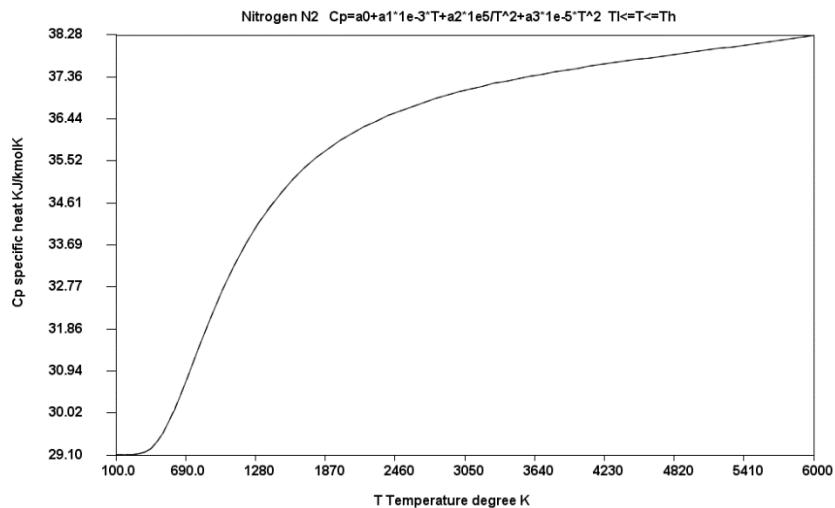


Figure 2.1 Cp of Nitrogen kJ/(kmolK)

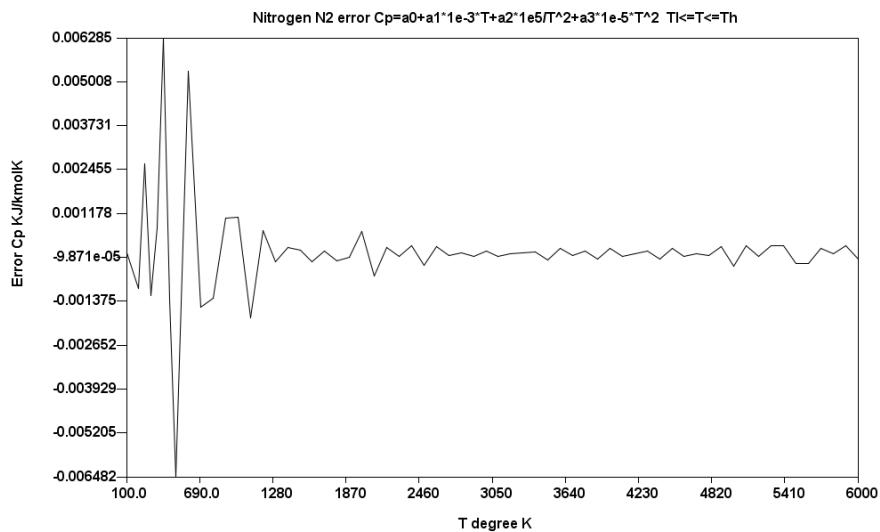


Figure 2.2 Error of Cp in Nitrogen kJ/(kmolK)

Table 2.2 Cp (kJ/kmol K) partial continuous curve fitting equations for N<sub>2</sub> Nitrogen

i	A <sub>i</sub>	B <sub>i</sub>	C <sub>i</sub>	D <sub>i</sub>	T <sub>li</sub> K	T <sub>hi</sub> K
0	29.4086307829	-2.2514470327	-0.0124732186	4.5208886188	100	350
1	27.6461690069	0.8823555268	0.7700742081	4.7644228675	350	700
2	21.6017064500	14.8784143146	3.8128084889	-4.1654669506	700	1200
3	29.8307659455	5.4215607907	-15.0430960215	-1.0896138268	1200	1700
4	35.4767415122	0.9735825946	-42.5476274875	-0.0974664401	1700	2200
5	34.9282028043	1.3194039653	-38.1841919451	-0.1599114820	2200	2700
6	36.2625256395	0.5815001033	-50.8983620805	-0.0457311313	2700	3200
7	35.6573409828	0.7661686027	-34.6659363416	-0.0598170521	3200	3700
8	36.4180454205	0.4325957723	-44.1847062013	-0.0201521727	3700	4200
9	38.0776880528	-0.1529603974	-80.3118075101	0.0367938172	4200	4700
10	37.7602843891	-0.0499492999	-73.1011559910	0.0277685442	4700	5200
11	39.9738552178	-0.8545553355	-77.5759376892	0.1012534665	5200	6000

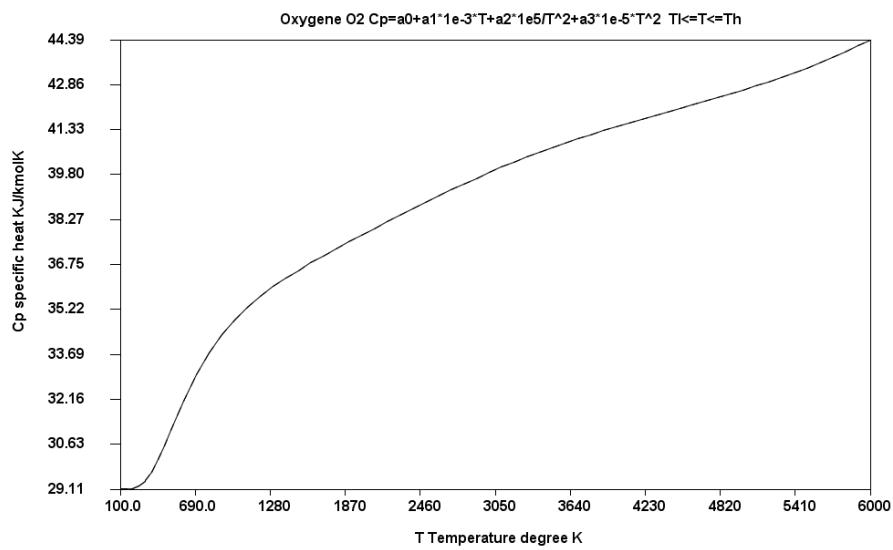


Figure 2.3 Cp of Oxygen kJ/(kmolK)

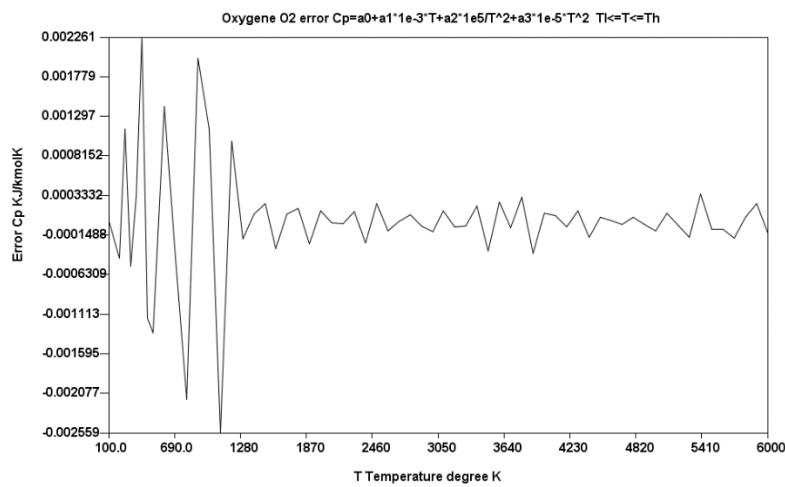


Figure 2.4 Error of Cp in Oxygen kJ/(kmolK)

Table 2.3 Cp (kJ/kmol K) partial continuous curve fitting equations for O<sub>2</sub> Oxygen

i	Ai	Bi	Ci	Di	Tli K	Thi K
0	30.4360497777	-11.2237599022	-0.0470904822	26.32148881	100	350
1	21.0013128464	23.6124140999	2.0465414905	-10.13517575	350	700
2	29.7425986041	7.9789108946	-6.1233332754	-2.24031495	700	1200
3	36.3127682664	0.0508140217	-19.6552390868	0.457041973	1200	1700
4	33.3454038017	2.3286053031	-4.5266442186	-0.037148052	1700	2200
5	31.0363625955	3.8048102300	11.9512729667	-0.301414132	2200	2700
6	29.7505550766	4.5049829956	24.8330382203	-0.408590258	2700	3200
7	32.1468550607	3.4665349136	-12.2598987205	-0.282718412	3200	3700
8	43.5270060418	-0.6697614318	-268.7947582132	0.140806012	3700	4200
9	54.7682266815	-4.2184956697	-604.9056827788	0.456500264	4200	4700
10	63.2608157288	-6.7264001385	-894.0814520230	0.664904416	4700	5200

11	71.3147995769	-8.8936565340	-1220.2917324495	0.828443569	5200	6000
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Table 2.4 Cp (kJ/kmol K) partial continuous curve fitting equations for Ar Argonne

i	Ai	Bi	Ci	Di	Tli K	Thi K
0	20.786	0.00E+00	0.00E+00	0.00E+00	100	6000

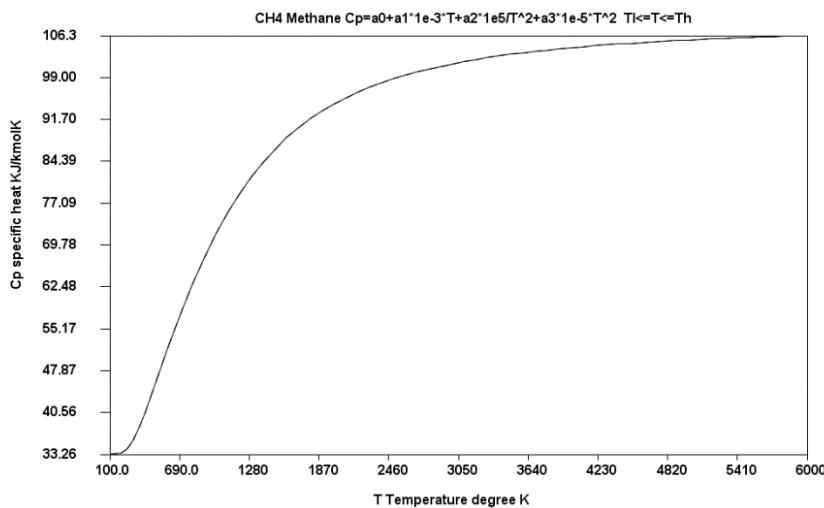


Figure 2.5 Cp of Methane kJ/(kmolK)

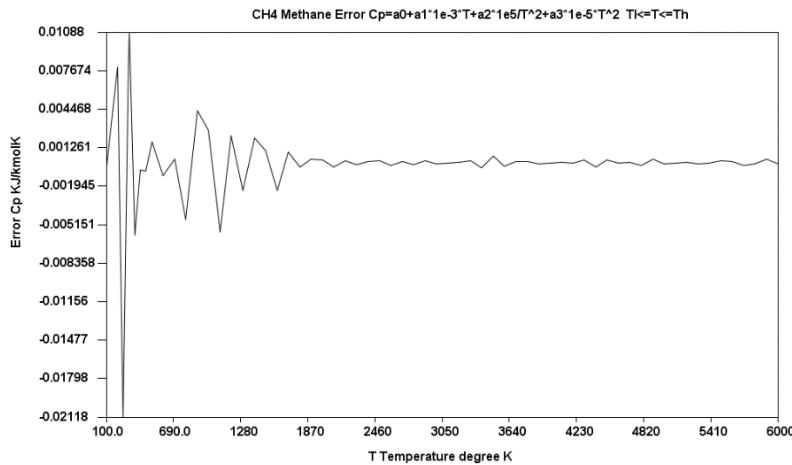


Figure 2.6 Error of Cp in Methane kJ/(kmolK)

Table 2.5 Cp (kJ/kmol K) partial continuous curve fitting equations for CH<sub>4</sub> Methane

i	Ai	Bi	Ci	Di	Tli K	Thi K
0	38.9123736504	-55.3874872142	-0.1624378115	150.9053147152	100	350
1	0.2017001921	103.1499706325	6.8244054323	-32.6647019325	350	700
2	11.5469381874	82.1956932915	-3.4532853748	-21.6049885321	700	1200
3	50.5726286784	38.2867248339	-96.5322057325	-7.6255082941	1200	1700
4	79.6812573368	14.8141853159	-230.3204126171	-2.2882055069	1700	2200
5	94.2035225106	5.8402863889	-343.8735028312	-0.7248723495	2200	2700
6	104.3572688164	0.9375681237	-473.8206804593	-0.0573582565	2700	3200
7	103.6301717202	1.3552795586	-469.7217838104	-0.1207924453	3200	3700

8	105.4581132228	0.6836699074	-509.3619167972	-0.0516420434	3700	4200
9	111.0178882010	-1.0114481078	-685.2166188470	0.0932934104	4200	4700
10	111.6872244691	-1.0507238460	-747.2766372898	0.0840648677	4700	5200
11	104.2567574929	0.8702029904	-413.4705005282	-0.0562039032	5200	6000

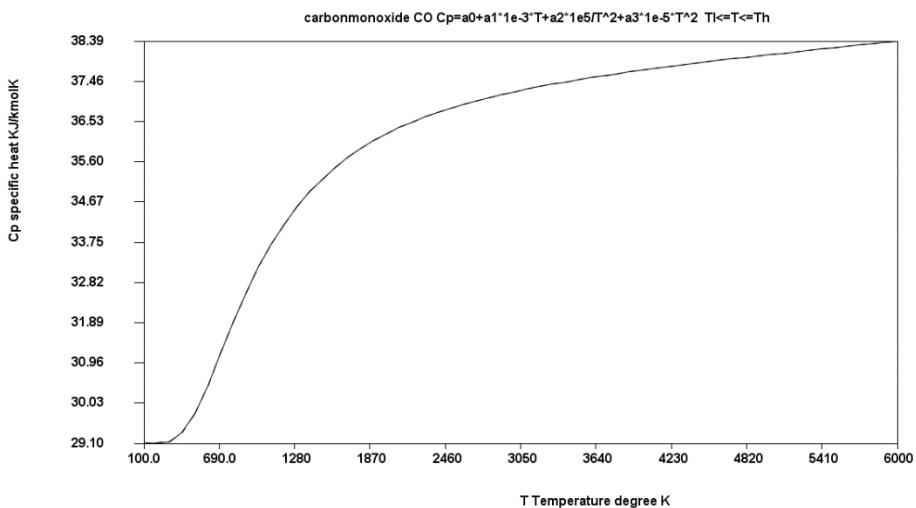


Figure 2.7 Cp of Carbonmonoxide kJ/(kmolK)

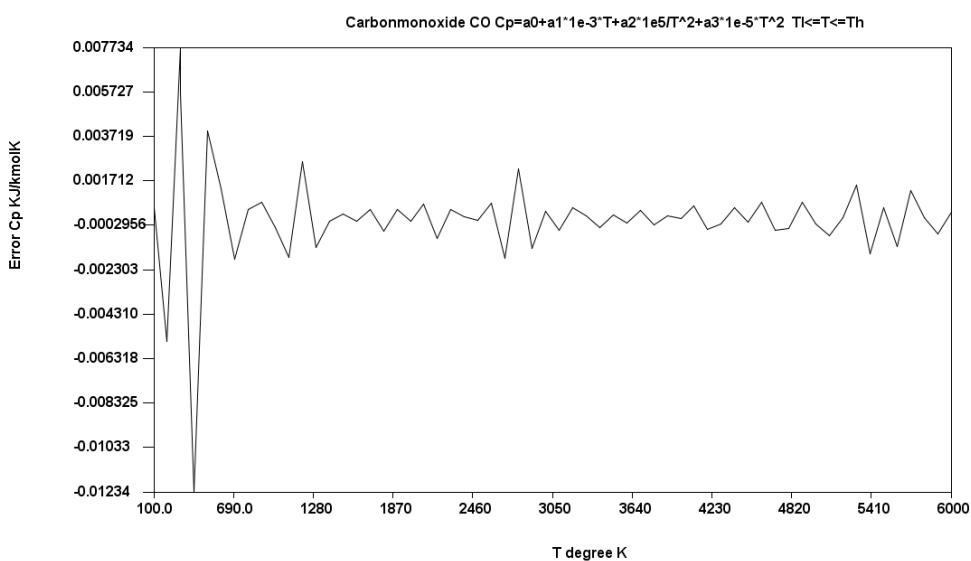


Figure 2.8 Error of Cp in Carbonmonoxide kJ/(kmolK)

Table 2.6 Cp coefficients of CO carbonmonoxide

i	Ai	Bi	Ci	Di	Tli K	Thi K
0	29.93325963	-5.92909	-0.03503	11.34129	100	500
1	20.96511337	17.1899	3.939062	-5.36552	500	1000
2	27.6978216	8.143888	-7.12349	-1.94678	1000	1500
3	34.48371707	1.797055	-30.749	-0.26475	1500	2000
4	35.28747409	1.206568	-34.6231	-0.14622	2000	2500
5	33.65691027	1.888256	-9.61422	-0.22202	2500	3000
6	41.22278785	-1.41581	-128.834	0.185886	3000	3500
7	57.90045043	-7.12289	-543.724	0.731525	3500	4000
8	32.21562692	1.829045	82.09653	-0.14561	4000	4500
9	70.47420068	-9.1211	-1308.69	0.737613	4500	5000

10	8.864569253	7.343303	1262.075	-0.50223	5000	5500
11	74.77026366	-8.58329	-2091.41	0.581296	5500	6000

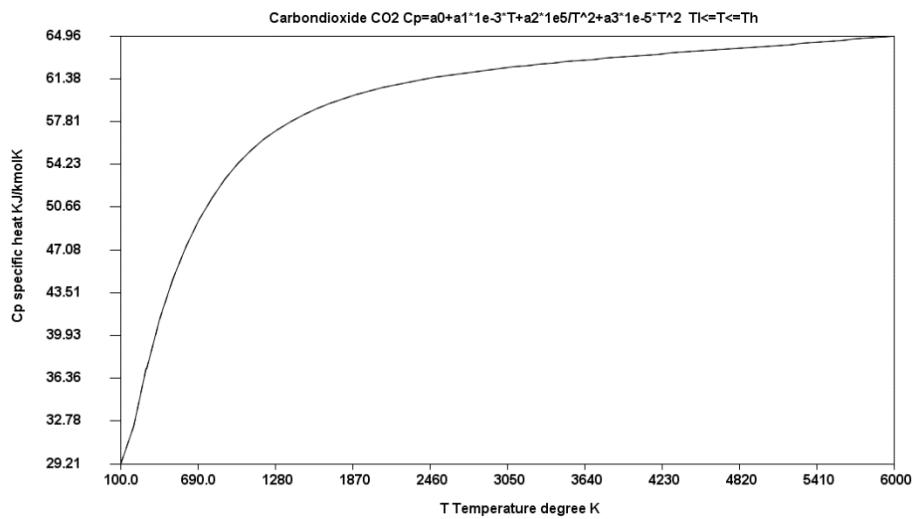


Figure 2.9 Cp of Carbondioxide kJ/(kmolK)

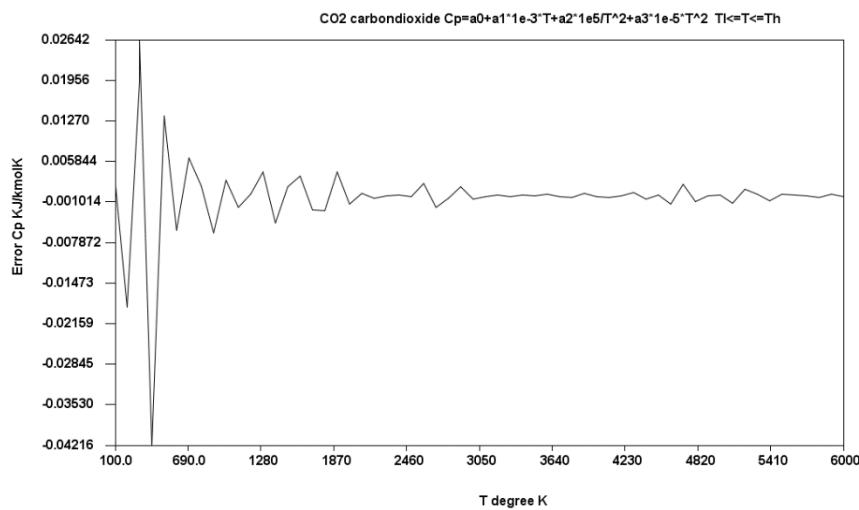


Figure 2.10 Error of Cp in Carbondioxide kJ/(kmolK)

Table 2.7 Cp coefficients of CO<sub>2</sub> carbondioxide

i	Ai	Bi	Ci	Di	Tli K	Thi K
0	17.3008737	80.30037	0.439594	-52.0538	100	500
1	32.82713013	33.27479	-5.03948	-11.2926	500	1000
2	47.39290533	12.63049	-27.1382	-3.00224	1000	1500
3	58.83520998	1.982446	-67.5235	-0.19015	1500	2000
4	60.89376216	0.909562	-86.5155	-0.04998	2000	2500
5	70.41645838	-3.8184	-201.416	0.611812	2500	3000
6	56.10341643	2.640738	16.60137	-0.22012	3000	3500
7	54.2839019	3.114683	80.13276	-0.24933	3500	4000
8	61.79621452	0.571644	-116.852	-0.00616	4000	4500
9	139.3004807	-21.3796	-2977.62	1.742156	4500	5000
10	-39.70675714	25.80975	4655.538	-1.75674	5000	5500
11	54.71006516	1.988445	271.9708	-0.06775	5500	6000

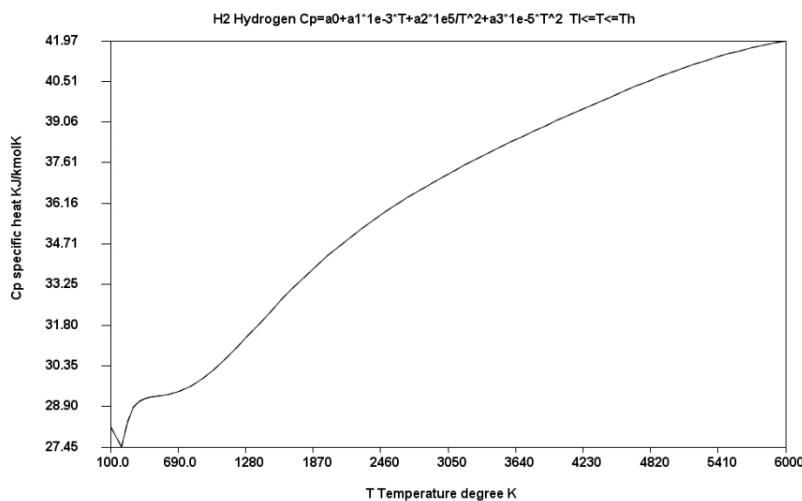


Figure 2.9 Cp of Hydrogen kJ/(kmolK)

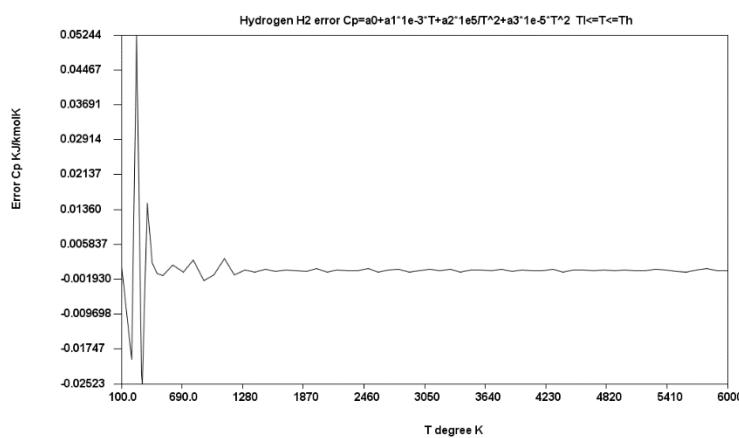


Figure 2.10 Error of Cp in Hydrogen kJ/(kmolK)

Table 2.8 Cp (kJ/kmol K) partial continuous curve fitting equations for H<sub>2</sub> Hydrogen

i	A <sub>i</sub>	B <sub>i</sub>	C <sub>i</sub>	D <sub>i</sub>	T <sub>li</sub> K	T <sub>hi</sub> K
0	14.7466359163	75.9881404127	0.6855650913	-104.7842726	100	350
1	31.8773831527	-7.0195707811	-1.2522956170	5.578010465	350	700
2	27.0099099507	1.4614189875	3.6026361193	1.374481793	700	1200
3	20.0497436420	9.8952098371	17.7745882168	-1.504385502	1200	1700
4	24.0484325285	6.8102257923	-2.3005034859	-0.832957652	1700	2200
5	29.2156479907	3.6508950604	-43.3579756078	-0.289222932	2200	2700
6	32.3142438373	2.0694485310	-78.1959957855	-0.062964072	2700	3200
7	35.6831488932	0.6762347250	-136.5501657830	0.099070362	3200	3700
8	35.5630278333	0.8914164594	-156.4897865015	0.060326987	3700	4200
9	21.7272706390	5.3732818954	234.0626600693	-0.347950527	4200	4700
10	13.0199056150	7.8723964969	547.1226571751	-0.549658083	4700	5200
11	6.7479064642	9.5439111926	806.6543827683	-0.674639418	5200	6000

Now we can establish other properties by using basic equations

$$ds = \frac{C_v(T)}{T} dT + \left( \frac{\partial P(T, v)}{\partial T} \right)_v dv \quad 2.33$$

$$ds = \frac{R - C_p(T)}{T} dT + \left( \frac{\partial P(T, v)}{\partial T} \right)_v dv \quad 2.34$$

$$du = C_v(T)dT + \left( T \left( \frac{\partial P(T,v)}{\partial T} \right)_v - P(T,v) \right) dv \quad 2.35$$

$$du = (R - C_p(T))dT + \left( T \left( \frac{\partial P(T,v)}{\partial T} \right)_v - P(T,v) \right) dv \quad 2.36$$

$$\left( \frac{\partial P}{\partial T} \right)_v = \frac{R}{v-b} - \frac{\partial}{\partial a} \left( \frac{a}{v^2+2bv-b^2} \right) \left( \frac{\partial a}{\partial T} \right)_v = \frac{R}{v-b} - \left( \frac{1}{v^2+2bv-b^2} \right) \left( \frac{\partial a}{\partial T} \right)_v \quad 2.37$$

$$s(T,V) = s_0 + \int_{T_0}^T \frac{R - C_p(T)}{T} dT + \int_{v_0}^v \left( \frac{\partial P}{\partial T} \right)_v dv \quad 2.38$$

$$u(T,v) = u_0 + \int_{T_0}^T (R - C_p(T))dT + \int_{v_0}^v \left( T \left( \frac{\partial P(T,v)}{\partial T} \right)_v - P(T,v) \right) dv \quad 2.39$$

$$h(T,v) = u + vP(T,v) \quad 2.40$$

$$g(T,v) = h(T,v) - TP(T,v) \quad 2.41$$

$$C_{pi}(T) = A_i + B_i 10^{-3}T + \frac{C_i 10^5}{T^2} + D_i 10^{-6}T^2 \quad T_{Li} \leq T \leq T_{Hi} \quad 2.42$$

$$C_p(T) = \sum_{i=0}^{n-1} y_i \quad C_{pi}(T) = \sum_{i=0}^{n-1} y_i \left[ A_i + B_i 10^{-3}T + \frac{C_i 10^5}{T^2} + D_i 10^{-6}T^2 \right] \quad T_{L0} \leq T \leq T_{H0} \quad 2.43$$

$$C_v(T) = R - C_p(T) = \sum_{i=0}^{n-1} y_i \quad C_{vi}(T) = \sum_{i=0}^{n-1} y_i [R - C_{pi}(T)] \quad 2.44$$

$$C_v(T) = \sum_{i=0}^{n-1} y_i \left[ R - \left( A_i + B_i 10^{-3}T + \frac{C_i 10^5}{T^2} + D_i 10^{-6}T^2 \right) \right] \quad 2.45$$

$$\int_{T_0}^T C_{vi}(T) dT = \left[ R(T - T_0) - \sum_{i=0}^{m-1} \left( A_i (T_{Hi} - T_{Li}) + \frac{B_i}{2} 10^{-3} (T_{Hi}^2 - T_{Li}^2) - C_i 10^5 \left( \frac{1}{T_{Hi}} - \frac{1}{T_{Li}} \right) + \frac{D_i 10^{-6}}{3} (T_{Hi}^3 - T_{Li}^3) \right) - \left( A_m (T - T_{Lm}) + \frac{B_m}{2} 10^{-3} (T^2 - T_{Lm}^2) - C_m 10^5 \left( \frac{1}{T} - \frac{1}{T_{Lm}} \right) + \frac{D_m 10^{-6}}{3} (T^3 - T_{Lm}^3) \right) \right] \quad 2.46$$

Where  $T_0 = T_{L0} = 100 K$  and  $T_{Lm} \leq T \leq T_{Hm}$

$$\int_{T_0}^T C_v(T) dT = \sum_{j=0}^{n-1} y_j \int_{T_0}^T C_{vj}(T) dT \quad 2.47$$

$$\int_{T_0}^T \frac{C_{vi}(T)}{T} dT = R \ln \left( \frac{T}{T_0} \right) - \left[ \sum_{i=0}^{m-1} A_i \ln \left( \frac{T_{Hi}}{T_{Li}} \right) + B_i (T_{Hi} - T_{Li}) - 2C_i 10^5 \left( \frac{1}{T_{Hi}^2} - \frac{1}{T_{Li}^2} \right) + \frac{D_m 10^{-6}}{2} (T_{Hi}^2 - T_{Li}^2) \right] - \left[ A_m \ln \left( \frac{T}{T_{Lm}} \right) + B_m (T - T_{Lm}) - 2C_m 10^5 \left( \frac{1}{T^2} - \frac{1}{T_{Lm}^2} \right) + \frac{D_m 10^{-6}}{2} (T^2 - T_{Lm}^2) \right] \quad 2.48$$

Where  $T_0 = T_{L0} = 100 K$  and  $T_{Lm} \leq T \leq T_{Hm}$

$$\int_{T_0}^T \frac{C_v(T) dT}{T} = \sum_{j=0}^{n-1} y_j \int_{T_0}^T \frac{C_{vj}(T) dT}{T} \quad 2.49$$

In the second approach air is taken as a single gas with pseudocritical properties. Specific heat values are taken from NIST tables. The same partial continuous specific heat equation is also used in here to calculate specific heat

$$C_{pi}(T) = A_i + B_i 10^{-3}T + \frac{C_i 10^5}{T^2} + D_i 10^{-6}T^2 \quad T_{Li} \leq T \leq T_{Hi} \quad \text{kJ/kg K} \quad 2.31$$

Curve-fitting coefficients are listed in Table 2.5

Enthalpy and entropy equations will be similar to each individual gas given in above equations. By using equations given above classes called air\_PR and air\_PR1 is developed. The result of this class is compared with IAPWS and perfect gas results which some sample calculations are given in table 2.6. It should be note that enthalpies of the perfect gas case (air\_PG) is the same regardless of temperature.

Table 2.9 Dry properties at different states for 4 dry air models

dry air model	P kPa	T deg. K	ro kg/m^3	h kJ/kg	s kJ/kgK
air IAPWS	100	300	1.1616	27.01362	0.098109
air PR	100	300	1.16016	26.96343	0.097905
air PR1	100	300	1.161866	27.03294	0.098201
air PG	100	300	1.161241	27.01008	0.098098
air IAPWS	300	300	3.486892	26.56149	-0.21858
air PR	300	300	3.47403	26.96343	-0.21692
air PR1	300	300	3.489305	27.07844	-0.21746
air PG	300	300	3.483724	27.01008	-0.21726
air IAPWS	500	300	5.814851	26.1112	-0.36654
air PR	500	300	5.779339	26.96343	-0.36302
air PR1	500	300	5.82155	27.12359	-0.36439
air PG	500	300	5.806207	27.01008	-0.36389
air IAPWS	1000	300	11.64547	24.99335	-0.56882
air PR	1000	300	11.50547	26.96343	-0.56066
air PR1	1000	300	11.67209	27.23498	-0.56407
air PG	1000	300	11.61241	27.01008	-0.56286
air IAPWS	100	350	0.995337	77.39375	0.253426
air PR	100	350	0.994555	77.28153	0.252992
air PR1	100	350	0.995496	77.44464	0.25363
air PG	100	350	0.99535	77.39021	0.253415
air IAPWS	200	350	1.990642	77.2332	0.053996
air PR	200	350	1.987528	77.28153	0.054252
air PR1	200	350	1.99128	77.49885	0.054621
air PG	200	350	1.9907	77.39021	0.054448
air IAPWS	300	350	2.985903	77.07313	-0.06285
air PR	300	350	2.978923	77.28153	-0.06191
air PR1	300	350	2.987341	77.55283	-0.06181
air PG	300	350	2.986049	77.39021	-0.06194
air IAPWS	500	350	4.976245	76.7544	-0.21041
air PR	500	350	4.956994	77.28153	-0.20808
air PR1	500	350	4.980244	77.66013	-0.20852
air PG	500	350	4.976749	77.39021	-0.20857
air IAPWS	1000	350	9.950525	75.96566	-0.41166
air PR	1000	350	9.874817	77.28154	-0.40592
air PR1	1000	350	9.966496	77.92432	-0.40766
air PG	1000	350	9.953498	77.39021	-0.40754
air IAPWS	100	450	0.773947	178.8411	0.508316
air PR	100	450	0.77368	178.6447	0.507616
air PR1	100	450	0.77402	178.9301	0.508614
air PG	100	450	0.774161	178.8375	0.508305
air IAPWS	300	450	2.320546	178.6841	0.192452
air PR	300	450	2.318168	178.6447	0.192614
air PR1	300	450	2.321214	179.1141	0.193362
air PG	300	450	2.322483	178.8375	0.192949
air IAPWS	500	450	3.865387	178.5285	0.045314
air PR	500	450	3.858841	178.6447	0.046337
air PR1	500	450	3.867254	179.2964	0.046836
air PG	500	450	3.870805	178.8375	0.046317
air IAPWS	1000	450	7.719584	178.1459	-0.15491
air PR	1000	450	7.693924	178.6447	-0.15175
air PR1	1000	450	7.727111	179.7449	-0.15186
air PG	1000	450	7.741609	178.8375	-0.15265

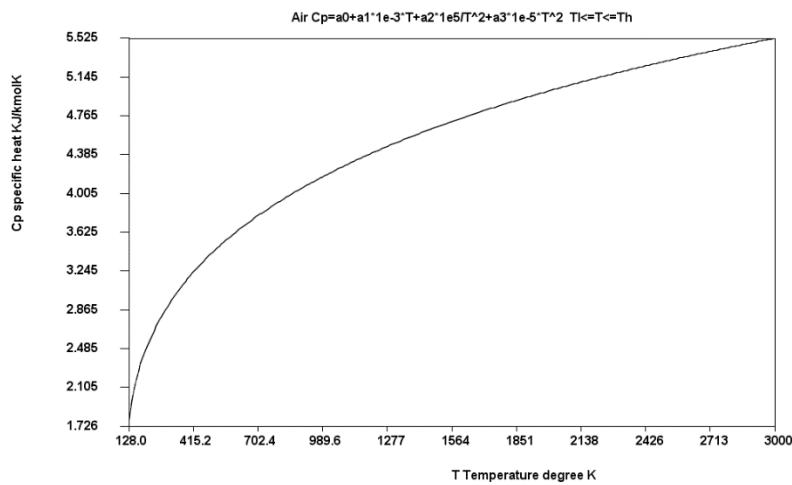


Figure 2.9 Cp of Air as a single gas kJ/(kgK)

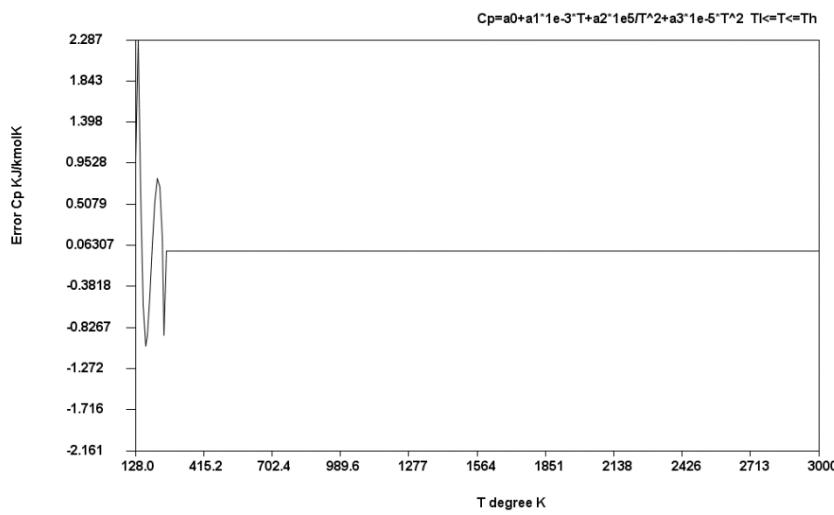


Figure 2.10 Error of Cp of air as a single gas kJ/(kmolK)

For the steam calculations class steamIAPWS\_IF97 is used. Details of these equation of state is as follows: International Association for the Properties of Water and Steam(IAPWS) is developed a new set of equation of states for industrial use. This new set of equations are developed in 1997. Steam properties are given by 5 sets of equation of states, as shown in the Figure 2.11

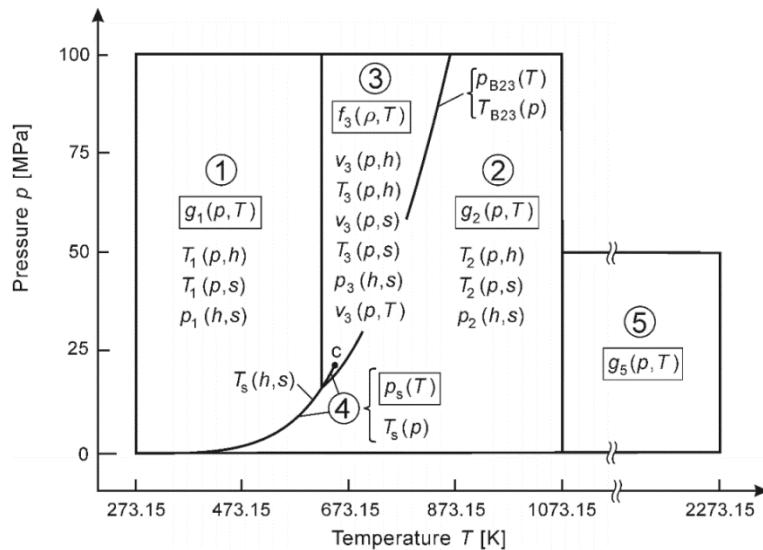


Figure 2.11 Zone definitions for different sub EOS for IAPWS-IF97

Figure 2.11 Equation of state regions for steam

The first equation, which covers basically liquid region has the following gibbs free energy form:

$$\frac{g_1(p, T)}{RT} = \gamma(\pi, \tau) = \sum_{i=1}^{34} n_i (71 - \pi)^{I_i} (\tau - 1222)^{J_i} \quad (2.50)$$

Where  $\pi = p / p^*$ ,  $\tau = T^* / T$ ,  $p^* = 16.62$  MPa and  $T^* = 1386$  K,  $R = 0.461526$  kJ/(kgK)

Table 2.10 coefficients of eqn. 2.50

i	I <sub>i</sub>	J <sub>i</sub>	n <sub>i</sub>	i	I <sub>i</sub>	J <sub>i</sub>	n <sub>i</sub>
1	0	-2	0.14632971213167	18	2	3	-4.4141845331E-06
2	0	-1	-0.84548187169114	19	2	17	-7.2694996298E-16

3	0	0	-3.75636036720400	20	3	-4	-3.1679644845E-05
4	0	1	3.38551691683850	21	3	0	-2.8270797985E-06
5	0	2	-0.95791963387872	22	3	6	-8.5205128120E-10
6	0	3	0.15772038513228	23	4	-5	-2.2425281908E-06
7	0	4	-0.01661641719950	24	4	-2	-6.5171222896E-07
8	0	5	0.00081214629984	25	4	10	-1.4341729938E-13
9	1	-9	0.00028319080124	26	5	-8	-4.0516996860E-07
10	1	-7	-0.00060706301566	27	8	-11	-1.2734301742E-09
11	1	-1	-0.01899006821842	28	8	-6	-1.7424871231E-10
12	1	0	-0.03252974877051	29	21	-29	-6.8762131296E-19
13	1	1	-0.02184171717541	30	23	-31	1.4478307829E-20
14	1	3	-0.00005283835797	31	29	-38	2.6335781663E-23
15	2	-3	-0.00047184321073	32	30	-39	-1.1947622640E-23
16	2	0	-0.00030001780793	33	31	-40	1.8228094581E-24
17	2	1	0.00004766139391	34	32	-41	-9.3537087292E-26

Thermodynamic relations can be calculated from these thermodynamic relations

$$\text{Specific volume: } v = \left( \frac{\partial g}{\partial p} \right)_T \quad (2.51)$$

$$\text{Specific enthalpy: } h = g - T \left( \frac{\partial g}{\partial p} \right)_P \quad (2.52)$$

$$\text{Specific internal energy: } u = g - T \left( \frac{\partial g}{\partial T} \right)_P - p \left( \frac{\partial g}{\partial p} \right)_T \quad (2.53)$$

$$\text{Specific entropy: } s = \left( \frac{\partial g}{\partial T} \right)_P \quad (2.54)$$

$$\text{Specific isobaric heat capacity: } C_p = \left( \frac{\partial h}{\partial T} \right)_P \quad (2.55)$$

$$\text{Specific isochoric heat capacity: } C_v = \left( \frac{\partial u}{\partial T} \right)_v \quad (2.56)$$

The second equation equation, which covers vapor region has the following gibbs free energy form:

$$\frac{g_2(p, T)}{RT} = \gamma(\pi, \tau) = \gamma^0(\pi, \tau) + \gamma^r(\pi, \tau) \quad (2.57)$$

Where  $\pi = p / p^*$ ,  $\tau = T^* / T$ ,  $R=0461526 \text{ kJ/(kgK)}$ ,  $\gamma^0(\pi, \tau)$  is the ideal gas part of EOS, and  $\gamma^r(\pi, \tau)$  is the real gas part of the EOS. Ideal gas part equation:

$$\gamma^0(\pi, \tau) = \ln \pi + \sum_{i=1}^9 n_i^0 \tau^{J_i} \quad (2.58)$$

Where  $p^* = 1 \text{ MPa}$  and  $T^* = 540 \text{ K}$

Table 2.11 coefficients of eqn. 2.58

i	J <sub>i</sub>	n <sub>i</sub> <sup>0</sup>	i	J <sub>i</sub>	n <sub>i</sub> <sup>0</sup>
1	0	-9.692768650E+00	6	-2	1.4240819171E+00
2	1	1.008665597E+01	7	-1	-4.3839511319E+00
3	-5	-5.608791128E-03	8	2	-2.8408632461E-01
4	-4	7.145273808E-02	9	3	2.1268463753E-02
5	-3	-4.071049822E-01			

dimensionless residual part of the basic equation  $g2(p, T)$  is as follows:

$$\gamma^r(\pi, \tau) = \sum_{i=1}^{43} n_i \pi^{I_i} (\tau - 0.5)^{J_i} \quad (2.60)$$

Where  $p^* = 1 \text{ MPa}$  and  $T^* = 540 \text{ K}$

Table 2.12 coefficients of eqn. 2.60

i	I <sub>i</sub>	J <sub>i</sub>	n <sub>i</sub>	i	I <sub>i</sub>	J <sub>i</sub>	n <sub>i</sub>
1	1	0		7	0		-5.9059564324270E-18
2	1	1	-1.7834862292358E-02	24	7	11	-1.2621808899101E-06
3	1	2	-4.5996013696365E-02	25	7	25	-3.8946842435739E-02
4	1	3	-5.7581259083432E-02	26	8	8	1.1256211360459E-11
5	1	6	-5.0325278727930E-02	27	8	36	-8.2311340897998E+00
6	2	1	-3.3032641670203E-05	28	9	13	1.9809712802088E-08
7	2	2	-1.8948987516315E-04	29	10	4	1.0406965210174E-19
8	2	4	-3.9392777243355E-03	30	10	10	-1.0234747095929E-13
9	2	7	-4.3797295650573E-02	31	10	14	-1.0018179379511E-09
10	2	36	-2.6674547914087E-05	32	16	29	-8.0882908646985E-11
11	3	0	2.0481737692309E-08	33	16	50	1.0693031879409E-01
12	3	1	4.3870667284435E-07	34	18	57	-3.3662250574171E-01
13	3	3	-3.2277677238570E-05	35	20	20	8.9185845355421E-25
14	3	6	-1.5033924542148E-03	36	20	35	3.0629316876232E-13
15	3	35	-4.0668253562649E-02	37	20	48	-4.2002467698208E-06
16	4	1	-7.8847309559367E-10	38	21	21	-5.9056029685639E-26
17	4	2	1.2790717852285E-08	39	22	53	3.7826947613457E-06
18	4	3	4.8225372718507E-07	40	23	39	-1.2768608934681E-15
19	5	7	2.2922076337661E-06	41	24	26	7.3087610595061E-29
20	6	3	-1.6714766451061E-11	42	24	40	5.5414715350778E-17
21	6	16	-2.1171472321355E-03	43	24	58	-9.4369707241210E-07
22	6	35	-2.3895741934104E+01				

Region 3 equation is given as Helmholtz free energy form:

$$\frac{f_3(\rho, T)}{RT} = \phi(\delta, \tau) = n_1 \ln \delta + \sum_{i=2}^{40} n_i \delta^{I_i} \tau^{J_i} \quad (2.61)$$

Where  $\delta = \rho / \rho^*$ ,  $\tau = T^* / T$ ,  $T^* = T_c = 647.096$  and  $R = 0461526 \text{ kJ/(kgK)}$

Table 2.13 coefficients of eqn. 2.61

i	I <sub>i</sub>	J <sub>i</sub>	n <sub>i</sub>	i	I <sub>i</sub>	J <sub>i</sub>	n <sub>i</sub>
1	0	0	1.065807002851E+00	21	3	4	-2.0189915023570E+00
2	0	0	-1.573284529024E+01	22	3	16	-8.2147637173963E-03
3	0	1	2.094439697431E+01	23	3	26	-4.7596035734923E-01
4	0	2	-7.686770787872E+00	24	4	0	4.3984074473500E-02
5	0	7	2.618594778795E+00	25	4	2	-4.4476435428739E-01
6	0	10	-2.808078114862E+00	26	4	4	9.0572070719733E-01
7	0	12	1.205336969652E+00	27	4	26	7.0522450087967E-01
8	0	23	-8.456681281250E-03	28	5	1	1.0770512626332E-01
9	1	2	-1.265431547771E+00	29	5	3	-3.2913623258954E-01
10	1	6	-1.152440780668E+00	30	5	26	-5.0871062041158E-01
11	1	15	8.852104398432E-01	31	6	0	-2.2175400873096E-02
12	1	17	-6.420776518161E-01	32	6	2	9.4260751665092E-02

13	2	0	3.849346018667E-01	33	6	26	1.6436278447961E-01
14	2	2	-8.521470882421E-01	34	7	2	-1.3503372241348E-02
15	2	6	4.897228154188E+00	35	8	26	-1.4834345352472E-02
16	2	7	-3.050261725697E+00	36	9	2	5.7922953628084E-04
17	2	22	3.942053687915E-02	37	9	26	3.2308904703711E-03
18	2	26	1.255840842431E-01	38	10	0	8.0964802996215E-05
19	3	0	-2.799932969871E-01	39	10	1	-1.6557679795037E-04
20	3	2	1.389979956946E+00	40	11	26	-4.4923899061815E-05

It should be noted that this set of equation is function of density and temperature, and basic equation is helmholts equation so, let us list definition of other thermodynamic properties

$$\text{Pressure: } p = \rho^2 \left( \frac{\partial f}{\partial \rho} \right)_T \quad (2.62)$$

$$\text{Specific enthalpy: } h = f - T \left( \frac{\partial f}{\partial T} \right)_P + \rho \left( \frac{\partial f}{\partial \rho} \right)_T \quad (2.63)$$

$$\text{Specific internal energy: } u = f - T \left( \frac{\partial f}{\partial T} \right)_P \quad (2.64)$$

$$\text{Specific entropy: } s = \left( \frac{\partial f}{\partial T} \right)_\rho \quad (2.65)$$

$$\text{Specific isobaric heat capacity: } C_p = \left( \frac{\partial h}{\partial T} \right)_P \quad (2.66)$$

$$\text{Specific isochoric heat capacity: } C_v = \left( \frac{\partial u}{\partial T} \right)_v \quad (2.67)$$

Region 4 of the equation defines saturation region. The basic equation is given as a polynomial

$$\beta^2 \vartheta^2 + n_1 \beta^2 \vartheta + n_2 \beta^2 + n_3 \beta \vartheta^2 + n_4 \beta \vartheta + n_5 \beta + n_6 \vartheta^2 + n_7 \vartheta + n_8 = 0 \quad (2.68)$$

Where

$$\beta = (p_s / p^*)^{0.25} \quad (2.69)$$

$$\vartheta = \frac{T_s}{T^*} + \frac{n_9}{(T_s/T^*) - n_{10}} \quad (2.70)$$

From this equation both saturation pressure and saturation temperature eqaution can be derived.

$$\frac{P_s}{P^*} = \left[ \frac{2C}{-B + (B^2 - 4AC)^{0.5}} \right]^4 \quad (2.71)$$

Where  $p^* = 1 \text{ MPa}$

$$A = \vartheta^2 + n_1 \vartheta + n_2$$

$$B = n_3 \vartheta^2 + n_4 \vartheta + n_5$$

$$C = n_6 \vartheta^2 + n_7 \vartheta + n_8$$

Table 2.14 coefficients of eqn. 2.71

i	n <sub>i</sub>	i	n <sub>i</sub>
1	1.1670521453E+03	6	1.4915108614E+01
2	-7.2421316703E+05	7	-4.8232657362E+03
3	-1.7073846940E+01	8	4.0511340542E+05
4	1.2020824702E+04	9	-2.3855557568E-01

5	-3.2325550322E+06	10	6.5017534845E+02
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It is also possible to derive saturation temperature equation from the basic polynomial as:

$$\frac{T_s}{T^*} = \frac{n_{10} + D - \left[ (n_{10} + D)^2 - 4(n_9 + n_{10}D) \right]^{0.5}}{2} \quad (2.72)$$

Where  $T^*=1$  K

$$D = \frac{2G}{-F - (F^2 - 4EG)^{0.5}}$$

$$E = \beta^2 + n_3\beta + n_6$$

$$F = n_1\beta^2 + n_4\beta + n_7$$

$$G = n_2\beta^2 + n_5\beta + n_8$$

And the final region for steam is region 5, again given as gibbs free equation type EOS

$$\frac{g_5(p, T)}{RT} = \gamma(\pi, \tau) = \gamma^0(\pi, \tau) + \gamma^r(\pi, \tau) \quad (2.73)$$

Where  $\pi = p / p^*$ ,  $\tau = T^* / T$ ,  $R=0461526$  kJ/(kgK),  $\gamma^0(\pi, \tau)$  is the ideal gas part of EOS, and

$\gamma^r(\pi, \tau)$  is the real gas part of the EOS. Ideal gas part equation:

$$\gamma^0(\pi, \tau) = \ln \pi + \sum_{i=1}^9 n_i^0 \tau^{J_i} \quad (2.74)$$

Where  $p^*=1$  MPa and  $T^*=1000$  K

Table 2.15 coefficients of eqn. 2.74

i	$J_i^0$	$n_i^0$	i	$J_i^0$	$n_i^0$
1	0	-13.1799836742	4	-2	0.3690153498
2	1	6.8540841634	5	-1	-3.1161318214
3	-3	-0.0248051489	6	2	-0.3296162654

The real gas part of the equation

$$\gamma^r(\pi, \tau) = \sum_{i=1}^{43} n_i \pi^{I_i} \tau^{J_i} \quad (2.75)$$

Table 2.16 coefficients of eqn. 2.75

i	$I_i$	$J_i$	$n_i$	i	$I_i$	$J_i$	$n_i$
1	1	1	1.5736404855E-03	4	2	3	2.2440037409E-06
2	1	2	9.0153761674E-04	5	2	9	-4.1163275453E-06
3	1	3	-5.0270077678E-03	6	3	7	3.7919454823E-08

At reference [8] more detailed set of equations are given for this calculations, for example reverse equation sets are given so that instead of additional curve fitting to get the variables this additional sets can be utilized. A program in java (steamIAPWS\_IF97) is prepared to use this set of EOS. A user interface is also prepared (steamTableIF97.java). The user interface input frame is given in the following figure.

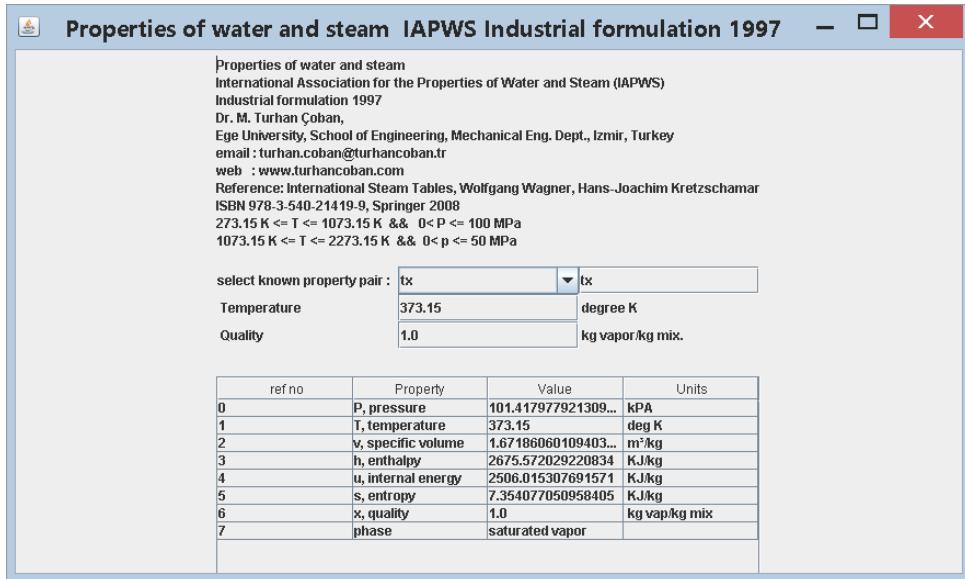


Figure 2.12 GUI for thermodynamic properties of steam

Equations for humid air:

$$h^{AV}(A, T, \rho) = (1 - A)h^V(T, \rho^V) + Ah^A(T, \rho^A) \quad 2.76$$

$$s^{AV}(A, T, \rho) = (1 - A)s^V(T, \rho^V) + As^A(T, \rho^A) \quad 2.77$$

### 3. ADDITIONAL FORMULATIONS

As a gas mixture, usually utilised in air conditioning industry, definitions used in standart air conditioning industry should also be expressed. Some of these properties are:

$$\text{Humidity ratio } w = \left(\frac{1}{A}\right) - 1 \quad 3.1$$

$$\text{Mole fraction of dry air } x_A = \frac{A(M_W/M_A)}{1-A[1-(M_W/M_A)]} \quad 3.2$$

$$\text{Mole fraction of water: } 1 - x_A \quad 3.3$$

$$\text{Mass fraction of dry air: } A = \frac{x_A}{1-(1-x_A)[1-(M_W/M_A)]} \quad 3.4$$

$$\text{Partial pressure of water: } P^W = (1 - x_A)P \quad 3.5$$

$$\text{Partial pressure of saturated water: } P^{Wsat} = (1 - x_A^{sat})P = P_\sigma(T) \quad 3.6$$

$$\text{Relative humidity: } RH = \frac{P^W}{P^{Wsat}} = \frac{(1-x_A)}{(1-x_A^{sat})} \quad 3.7$$

$$\text{Degree of saturation DOS} = \frac{m^W}{m^{Wsat}} = \frac{1-A}{1-A^{sat}} \quad 3.8$$

$$\text{Dew point temperature (saturation temperature at partial pressure of water) } T_\sigma(P^W) = T_\sigma((1 - x_A)P) \quad 3.9$$

Another concept used for wet air is adyabatic saturation temperature. If air flow through an infinite length channel filled with water at the bottom and all walls are insulated, it will absorb water and will be reached to adyabatic saturation point. The temparature of adyabatic saturation point is also called wet air temperature, it is an idealised thermodynamic concept and can be calculated from the energy balance of the infinitely long channel. Basic energy equation:

Energy of the air entering the channel = enegy of the air leaving the channel + energy of evaporated water,

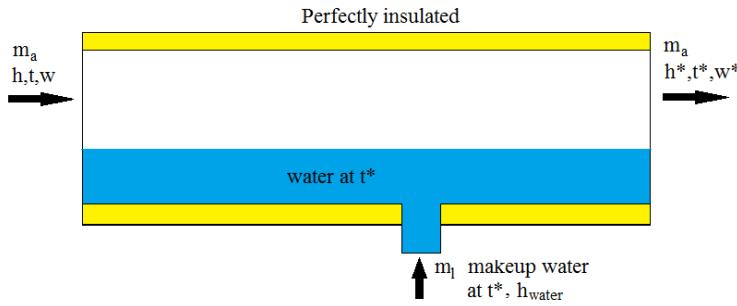


Figure 2.1 Adiabatic saturation temperature

$$So \ m_a h(A, T, P) + m_l h_{water}^*(T^*, P) = m_a h(A^*, T^*, P) \quad 3.10$$

$$m_l = m_a(w^* - w) = m_a \left( \frac{1}{A^*} - \frac{1}{A} \right) \quad 3.11$$

$$h(A, T, P) + \left( \frac{1}{A^*} - \frac{1}{A} \right) h_{water}^*(T^*, P) = h(A^*, T^*, P) \quad 3.12$$

Since the exit is at saturation state relative humidity at exit is 1 so

$$RH = \frac{P^W}{P^{Wsat}} = \frac{(1-x_A)}{(1-x_A^{sat})} = 1.0 \quad 3.13$$

Solution of these equations will require root finding methods.

In order to calculate thermophysical properties (thermal conductivity and viscosity) of humid water Kadoya et al[1] equations are used. This equations has the following form:

$$\eta_0(T_r) = A_0 T_r + A_1 T_r^{0.5} + A_2 + \frac{A_3}{T_r} + \frac{A_4}{T_r^2} + \frac{A_5}{T_r^3} + \frac{A_6}{T_r^4} \quad 3.14$$

$$\Delta\eta(\rho_r) = \sum_{i=1}^4 B_i \rho_r^i \quad 3.15$$

$$\eta(T_r, \rho_r) = H[\eta_0(T_r) + \Delta\eta(\rho_r)] \quad 3.16$$

$$k_0(T_r) = C_0 T_r + C_1 T_r^{0.5} + C_2 + \frac{C_3}{T_r} + \frac{C_4}{T_r^2} + \frac{C_5}{T_r^3} + \frac{C}{T_r^4} \quad 3.17$$

$$\Delta k(\rho_r) = \sum_{i=1}^4 D_i \rho_r^i \quad 3.18$$

$$k(T_r, \rho_r) = \Delta[k_0(T_r) + \Delta k(\rho_r)] \quad 3.19$$

$$Where \rho_r = \rho/\rho^* \quad T_r = T/T^* \quad 3.20$$

Coefficients of the equations are given in Table 3.1

### Coefficients of equations 3.14-3.19

$T^* = 132.5 \text{ K}$	$\rho^* = 314.3 \text{ kg/m}^3$	$\Lambda = 25.9778 (10^{-3} W/(mK))$	$H=6.1609 (10^{-6} \text{ Pas})$	
i	A <sub>i</sub>	B <sub>i</sub>	C <sub>i</sub>	D <sub>i</sub>
0	0.128517	0.465601	0.239503	0.402287
1	2.60661	1.26469	0.00649768	0.356603
2	-1	-0.511425	1	-0.163159
3	-0.709661	0.2746	-1.92615	0.138059
4	0.662534		2.00383	-0.0201725
5	-0.197846		-1.07553	
6	0.00770147		0.229414	

In order to both check and error control purposes a surface cubic spline curve fitting equivalent formulation is also created. The data for curve fitting based on NIST data for thermal conductivity and viscosity for dry air. Viscosity and thermal conductivity values are taken from IAPWS Industrial Formulation 1997[15]. This equations are as follows:

Viscosity equations:

$$\eta(\rho, T) = \psi(\delta, \theta) = \eta^* [\psi_0(\theta) \psi_1(\delta, \theta)] \quad 3.21$$

$$\text{Where } \eta^* = 10^{-6} \text{ Pas} \quad \delta = \frac{\rho}{\rho^*} \quad \theta = T/T^*$$

$$\text{with } T^* = T_c = 647.096 \text{ K} \quad \rho^* = \rho_c = 322 \text{ kg/m}^3$$

$$\psi_0(\theta) = \theta^{0.5} \left[ \sum_{i=1}^4 n_i^0 \theta^{1-i} \right]^{-1} \quad 3.22 \text{ Coefficients of equation given below:}$$

Table 3.2 Coefficients of equation 3.22

i	$n_i^0$
1	0.167752e-1
2	0.220462e-1
3	0.6366564e-2
4	-0.241605e-2

$$\psi_1(\delta, \theta) = \exp \left[ \delta \sum_{i=1}^{21} n_i (\delta - 1)^{I_i} \left( \frac{1}{\theta} - 1 \right)^{J_i} \right] \quad 3.23$$

Table 3.3 Coefficients of equation 3.23

i	I <sub>i</sub>	J <sub>i</sub>	N <sub>i</sub>	i	I <sub>i</sub>	J <sub>i</sub>	N <sub>i</sub>
1	0	0	5.200940E-01	12	2	2	-7.724790E-01
2	0	1	8.508950E-02	13	2	3	-4.898370E-01
3	0	2	-1.083740E+00	14	2	4	-2.570400E-01
4	0	3	-2.895550E-01	15	3	0	1.619130E-01
5	1	0	2.225310E-01	16	3	1	2.573990E-01
6	1	1	9.991150E-01	17	4	0	-3.253720E-02
7	1	2	1.887970E+00	18	4	3	6.984520E-02
8	1	3	1.266130E+00	19	5	4	8.721020E-03
9	1	5	1.205730E-01	20	6	3	-4.356730E-03
10	2	0	-2.813780E-01	21	6	5	-5.932640E-04
11	2	1	-9.068510E-01				

Thermal conductivity equations

$$\frac{k(\rho, T)}{\lambda^*} = \Lambda(\delta, \theta) = \Lambda_0(\theta) + \Lambda_1(\delta) + \Lambda_2(\delta, \theta) \quad 3.24$$

$$\Lambda_0(\theta) = \theta^{0.5} \sum_{i=1}^4 n_i^0 \theta^{i-1} \quad 3.25$$

Table 3.4 Coefficients of equation 3.25

i	$n_i^0$
1	0.102811e-1
2	0.299621e-1
3	0.156146e-1
4	-0.422464e-2

$$\Lambda_1(\delta) = n_1 + n_2\delta + n_3 \exp[n_4(\delta + n_5)^2] \quad 3.26$$

Table 3.5 Coefficients of equation 3.26

i	$n_i$
1	0.39707
2	0.400302
3	-0.171587e4
4	-0.239219e1

$$\Lambda_2(\delta, \theta) = (n_1 \theta^{-10} + n_2) \delta^{1.8} \exp[n_2(1 - \delta^{2.8})] + n_4 A \delta^B \exp\left[\left(\frac{B}{1+B}\right)(1 - \delta^{1+B})\right] +$$

$$n_5 \exp[n_6 \theta^{1.5} + n_7 \delta^{-5}] \quad 3.27$$

$$A(\theta) = 2 + n_8(\Delta\theta)^{-0.6} \quad 3.27a$$

$$B(\theta) = \begin{cases} (\Delta\theta)^{-1} & \text{for } \theta \geq 1 \\ n_9(\Delta\theta)^{-0.6} & \text{for } \theta < 1 \end{cases} \quad 3.27b \quad \text{with } \Delta\theta = |\theta - 1| + n_{10}$$

Table 3.6 Coefficients of equation 3.27

i	$n_i$	i	$n_i$
1	7.013090E-02	6	-4.117170E+00
2	1.185200E-02	7	-6.179370E+00
3	6.428570E-01	8	8.229940E-02
4	1.699370E-03	9	1.009320E+01
5	-1.020000E+00	10	3.089760E-03

For mixing of dry air and water Wilke equation[6] will be used. This equation has the following form:

$$\eta_{mix} = \frac{x_A \eta_A}{x_A + \eta_A \phi_{AV}} + \frac{(1-x_A) \eta_V}{(1-x_A) + \eta_V \phi_{VA}} \quad 3.28 \quad \text{where}$$

$$\phi_{AV} = \frac{\left[1 + (\eta_A / \eta_V)^{1/2} (M_V / M_A)^{1/4}\right]}{\{8[1 + (M_A / M_V)]\}^{1/2}} \quad 3.28a$$

$$\phi_{VA} = \phi_{AV} \eta_V / \eta_A (M_A / M_V) \quad 3.28b$$

For thermal conductivity, similar equations will be used.

$$k_{mix} = \frac{x_A k_A}{x_A + k_A \phi_{AV}} + \frac{(1-x_A) k_V}{(1-x_A) + k_V \phi_{VA}} \quad 3.29 \quad \text{where}$$

$$\phi_{AV} = \frac{\left[1 + (k_A / k_V)^{1/2} (M_V / M_A)^{1/4}\right]}{\{8[1 + (M_A / M_V)]\}^{1/2}} \quad 3.29a$$

$$\phi_{VA} = \phi_{AV} k_V / k_A (M_A / M_V) \quad 3.29b$$

#### 4. COMPUTER DEVELOPMENT AND ERROR CHECK

Several programs (classes) in java language to carry out this analysis. The list of program(class) names are given in Table 4.1

Table 4.1 Program lists

Class/interface name	Source
Interface if_x	Base interface general definition for function f(x)
Interface if_x	Base interface general definition for function f0(x0,x1,x2..) f1(x0,x1,x2...) ...
steam	Water-steam EOS Keenan, Keys,Hill, Moore 1969
steamIAPWS_IF97	Water-steam EOS International Steam Tables, Wolfgang Wagner, Hans-Joachim Kretzschmar ISBN 978-3-540-21419-9
steam_IAPWS95	The IAPWS Formulation 1995 for the Thermodynamic Properties of Ordinary Water Substance for General and Scientific Use W. Wagner and A. PruB J. Phys. Chem. Ref. Data, Vol. 31, No. 2, 2002
air_IAPWS	Dry air EOS. Guideline on an Equation of State for Humid Air in Contact with Seawater and Ice, Consistent with the IAPWS Formulation 2008 for the Thermodynamic Properties of Seawater
air_PR	Dry air EOS. using Peng Robinson EOS for mixtures The properties of Gases & Liquids Robert C. Reid et al., Janaf Tables (NIST data <a href="https://janaf.nist.gov/">https://janaf.nist.gov/</a> )
air-PR1	Dry air EOS. using Peng Robinson EOS as a single gas. Pseudocritical properties are assumed for air. The properties of Gases & Liquids Robert C. Reid et al., Janaf Tables (NIST data <a href="https://janaf.nist.gov/">https://janaf.nist.gov/</a> )
Air_PG	Dry air EOS. using Perfect gas EOS as a single gas.
humid_air_IAPWS	Humid air EOS, Guideline on an Equation of State for Humid Air in Contact with Seawater and Ice, Consistent with the IAPWS Formulation 2008 for the Thermodynamic Properties of Seawater
humid_air_PR	Humid air EOS using peng Robinson EOS as mixtures fro dry air, The properties of Gases & Liquids Robert C. Reid et al., Janaf Tables (NIST data <a href="https://janaf.nist.gov/">https://janaf.nist.gov/</a> , <a href="http://www.turhancoban.com">www.turhancoban.com</a> )
humid_air_PR1	Humid air EOS using peng Robinson EOS as a single gas for dry air, The properties of Gases & Liquids Robert C. Reid et al., Janaf Tables (NIST data <a href="https://janaf.nist.gov/">https://janaf.nist.gov/</a> , <a href="http://www.turhancoban.com">www.turhancoban.com</a> )
humid_air_PG	Humid air EOS using Perfect gas EOS for air, The properties of Gases & Liquids Robert C. Reid et al., Janaf Tables (NIST data <a href="https://janaf.nist.gov/">https://janaf.nist.gov/</a> , <a href="http://www.turhancoban.com">www.turhancoban.com</a> )
Gas	Pure ideal gas EOS including air and other gases <a href="http://www.turhancoban.com">www.turhancoban.com</a> _SCO1.jar
Plot	2D Plot program <a href="http://www.turhancoban.com">www.turhancoban.com</a> _SCO1.jar
GasTable	Graphic User Interface for class Gas
Wetair	Ideal Gas Humid air EOS utilises Gas class and steam class
pST	Graphic User Interface for class wetair
steamTable	Graphic User Interface for class steam
steamTableIF97	Graphic User Interface for class steamIAPWS_IF97
steamTableIAPWS95	Graphic User Interface for class steam_IAPWS95

In Table 4.2 different EOS results compared for 3 thermodynamic states. A Graphic user interface programs are also given for non-researchers to utilise these programs. For utilisation of researchers a small sample code is given to show calling of thermodynamic and thermophysical properties for a given state.

Table 4.2 Comparisons of 3 different EOS for water & Steam thermodynamic properties.

Class	P kPa	T degree K	v m <sup>3</sup> /kg	h kJ/kg	u kJ/kg	s kJ/(kgK)	x kg vap/kg
steam	3.535746	300	19.55028	1331.584	1262.459	4.455994	0.5
steamIAPWS_IF97	3.536589	300	19.54153	1331.234	1262.124	4.45533	0.5

steam_IAPWS95	3.536718	300	19.54013	1331.21	1262.102	4.455244	0.5
steam	101.325	300	0.001003	112.7105	112.6089	0.393245	-1.1E-09
steamIAPWS_IF97	101.325	300	0.001003	112.665	112.5634	0.393097	-1
steam_IAPWS95	101.325	300	0.001003	112.6549	112.5532	0.393062	-2
steam	101.325	400	1.801666	2730.178	2547.624	7.495023	2.467381
steamIAPWS_IF97	101.325	400	1.802056	2730.272	2547.679	7.496078	2
steam_IAPWS95	101.325	400	1.801984	2730.301	2547.715	7.496202	2

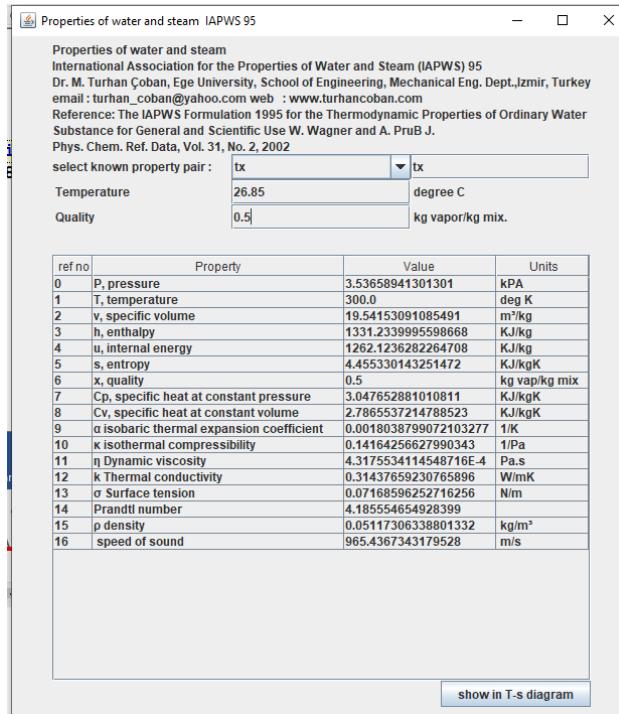


Figure 4.1 steam\_IAPWS95 Graphic user interface

By using equations given above a class called air\_IAPWS95 is developed. The result of this class is compared with Peng-Robinson EOS results as a mixing of the gases inside air and as a single gas with pseudocritical properties plus ideal gas equation of state. Results are given in table 4.3. It should be note that enthalpies of the perfect gas case (air\_PG) is the same regardless of temperature.

Table 4.3 Air properties at different states for 4 dry air models

dry air model	P kPa	T deg. K	ro kg/m^3	h kJ/kg	s kJ/kgK
air_IAPWS	100	300	1.1616	27.01362	0.098109
air_PR	100	300	1.16016	26.96343	0.097905
air_PR1	100	300	1.161866	27.03294	0.098201
air_PG	100	300	1.161241	27.01008	0.098098
air_IAPWS	300	300	3.486892	26.56149	-0.21858
air_PR	300	300	3.47403	26.96343	-0.21692
air_PR1	300	300	3.489305	27.07844	-0.21746
air_PG	300	300	3.483724	27.01008	-0.21726
air_IAPWS	500	300	5.814851	26.1112	-0.36654
air_PR	500	300	5.779339	26.96343	-0.36302
air_PR1	500	300	5.82155	27.12359	-0.36439
air_PG	500	300	5.806207	27.01008	-0.36389
air_IAPWS	1000	300	11.64547	24.99335	-0.56882
air_PR	1000	300	11.50547	26.96343	-0.56066
air_PR1	1000	300	11.67209	27.23498	-0.56407
air_PG	1000	300	11.61241	27.01008	-0.56286

air IAPWS	100	350	0.995337	77.39375	0.253426
air PR	100	350	0.994555	77.28153	0.252992
air PR1	100	350	0.995496	77.44464	0.25363
air PG	100	350	0.99535	77.39021	0.253415
air IAPWS	200	350	1.990642	77.2332	0.053996
air PR	200	350	1.987528	77.28153	0.054252
air PR1	200	350	1.99128	77.49885	0.054621
air PG	200	350	1.9907	77.39021	0.054448
air IAPWS	300	350	2.985903	77.07313	-0.06285
air PR	300	350	2.978923	77.28153	-0.06191
air PR1	300	350	2.987341	77.55283	-0.06181
air PG	300	350	2.986049	77.39021	-0.06194
air IAPWS	500	350	4.976245	76.7544	-0.21041
air PR	500	350	4.956994	77.28153	-0.20808
air PR1	500	350	4.980244	77.66013	-0.20852
air PG	500	350	4.976749	77.39021	-0.20857
air IAPWS	1000	350	9.950525	75.96566	-0.41166
air PR	1000	350	9.874817	77.28154	-0.40592
air PR1	1000	350	9.966496	77.92432	-0.40766
air PG	1000	350	9.953498	77.39021	-0.40754
air IAPWS	100	450	0.773947	178.8411	0.508316
air PR	100	450	0.77368	178.6447	0.507616
air PR1	100	450	0.77402	178.9301	0.508614
air PG	100	450	0.774161	178.8375	0.508305
air IAPWS	300	450	2.320546	178.6841	0.192452
air PR	300	450	2.318168	178.6447	0.192614
air PR1	300	450	2.321214	179.1141	0.193362
air PG	300	450	2.322483	178.8375	0.192949
air IAPWS	500	450	3.865387	178.5285	0.045314
air PR	500	450	3.858841	178.6447	0.046337
air PR1	500	450	3.867254	179.2964	0.046836
air PG	500	450	3.870805	178.8375	0.046317
air IAPWS	1000	450	7.719584	178.1459	-0.15491
air PR	1000	450	7.693924	178.6447	-0.15175
air PR1	1000	450	7.727111	179.7449	-0.15186
air PG	1000	450	7.741609	178.8375	-0.15265

Properties of steam and dry air combined according to equations given above sections. The program is called humid\_air\_PG for mixed dry air and humid\_air\_PG1 for dry air as a single gas.

Properties of humid air Cubic Peng-Robinson EOS based. Air as a gas mixture of component gases					
array no	property name	value humid air mass fr base	unit	value dry air mass fr base	unit
0	P, pressure	100.0	kPa		
1	T, temperature	26.850000000000023	deg C		
2	v, specific volume	0.879209904366791	m <sup>3</sup> /kg humid air	0.8880020034104591	m <sup>3</sup> /kg dry air
3	h, enthalpy	51.95147960295802	KJ/kg	52.47099439898761	kJ/(kg dry air)
4	s, entropy	0.1849688167931825	KJ/kgK	0.18681850496125146	kJ/(kg dry air K)
5	w, specific humidity, humidity ratio	0.01000000000000122	kg vap/kg dry air		
6	A, mass fraction of dry air	0.99009900990099	kg dry air/kg humid air		
7	xA, mole fraction of dry air	0.9841761363142828	kmol dry air/kmol humi...		
8	relative humidity	0.4474166596195414			
9	Adiabatic saturation temperature	18.51341603567795	degree C		
10	dew point temperature	13.840444902461059	degree C		
11	η Dynamic viscosity	4.4811104903645E-4	Pa.s		
12	k Thermal conductivity	0.02767903995508192	W/mK		
13	degree of saturation	0.44409115606801813			
14	Pa dry air partial pressure	98.41761363142828	kPa		
15	Pv water moisture partial pressure	1.5823863685717199	kPa		
16	ρ density	1.1373848213416138	kg humid air/m <sup>3</sup>	1.1261235854867462	kg dry air/m <sup>3</sup>
17	Cp isochoric specific heat	1.0362836981699992	kJ/(kg humid air K)	1.0466465351516994	kJ/(kg dry air K)
18	Cv isovolumetric specific heat	0.7246574099112448	kJ/(kg humid air K)	0.7319039840103574	kJ/(kg dry air K)
19	ha enthalpy of dry air KJ/kg dry air	26.9633529821748	kJ/(kg humid air )	27.23298651199655	kJ/(kg dry air K)
20	hv enthalpy of vapor KJ/kg water	2550.7641416812485	kJ/(kg water )		

Figure 4.1 humid\_air\_PR (Air as a gas mixture) Graphic output for P=100 kPa and T=300 K

array no	property name	value humid air mass fr ba...	unit	value dry air mass fr base	unit
0	P, pressure	200.0	kPa		
1	T, temperature	26.850000000000023	deg C		
2	v, specific volume	0.440008996131872	m³/kg humid air	0.44440908960931913	m³/kg dry air
3	h, enthalpy	51.944300181523914	KJ/kg	52.46374318333916	kJ/(kg dry air)
4	s, entropy	-0.014956640248611	KJ/kgK	-0.015106230665109714	kJ/(kg dry air K)
5	w, specific humidity, hu...	0.01000000000000122	kg vap/kg dry air		
6	A, mass fraction of dry air	0.99009900990099	kg dry air/kg humid air		
7	xA, mole fraction of dry air	0.9841761363142828	kmol dry air/kmol humid...		
8	relative humidity	0.8948333192390828			
9	Adiabatic saturation tem...	26.83999999999918	degree C		
10	dew point temperature	24.973249865022353	degree C		
11	η Dynamic viscosity	4.482400947149639E-4	Pa.s		
12	k Thermal conductivity	0.0277196086709372	W/mK		
13	degree of saturation	0.8942004153001419			
14	Pa dry air partial pressure	196.83522726285656	kPa		
15	Pv water moisture partial...	3.1647727371434398	kPa		
16	ρ density	2.2726807880727486	kg humid air/m³	2.25017899809183	kg dry air/m³
17	Cp isochoric specific heat	1.036280900379695	kJ/(kg humid air K)	1.046643709383492	kJ/(kg dry air K)
18	Cv isovolumetric specific...	0.7247672872815254	kJ/(kg humid air K)	0.7320149601543408	kJ/(kg dry air K)
19	ha enthalpy of dry air KJ...	26.963353387901307	kJ/(kg humid air )	27.232986921780324	kJ/(kg dry air K)
20	hv enthalpy of vapor KJ...	2550.038979543754	kJ/(kg water )		

Figure 4.2 humid\_air\_PR(Air as a gas mixture) Graphic output for P=200 kPa and T=300 K

array no	property name	value humid air mass fr ...	unit	value dry air mass fr base	unit
0	P, pressure	100.0	kPa		
1	T, temperature	26.850000000000023	deg C		
2	v, specific volume	0.8779341043871022	m³/kg humid air	0.8867134454309733	m³/kg dry air
3	h, enthalpy	52.02002462463205	KJ/kg	52.540224870878376	kJ/(kg dry air)
4	s, entropy	0.1852597688313493	KJ/kgK	0.18711236651966281	kJ/(kg dry air K)
5	w, specific humidity, h...	0.01000000000000122	kg vap/kg dry air		
6	A, mass fraction of dry...	0.99009900990099	kg dry air/kg humid air		
7	xA, mole fraction of dr...	0.984176179326808	kmol dry air/kmol hu...		
8	relative humidity	0.44741544434487798			
9	Adiabatic saturation te...	18.519809229567613	degree C		
10	dew point temperature	13.840403083643821	degree C		
11	η Dynamic viscosity	4.481108595837139E-4	Pa.s		
12	k Thermal conductivity	0.027679038036385732	W/mK		
13	degree of saturation	0.44408995687375685			
14	Pa dry air partial pressure	98.4176179326808	kPa		
15	Pv water moisture parti...	1.582382067319199	kPa		
16	ρ density	1.1390376510069782	kg humid air/m³	1.1277600505019585	kg dry air/m³
17	Cp isochoric specific h...	1.0377745914519034	kJ/(kg humid air K)	1.0481523373664225	kJ/(kg dry air K)
18	Cv isovolumetric speci...	0.7261476481858238	kJ/(kg humid air K)	0.7334091246676822	kJ/(kg dry air K)
19	ha enthalpy of dry air ...	27.032583435393093	kJ/(kg humid air )	27.302909269747026	kJ/(kg dry air K)
20	hv enthalpy of vapor ...	2550.7641435484975	kJ/(kg water )		

array no	property name	value humid air mass fr ...	unit	value dry air mass fr base	unit
0	P, pressure	100.0	kPa		
1	T, temperature	26.850000000000023	deg C		
2	v, specific volume	0.8779341043871022	m³/kg humid air	0.8867134454309733	m³/kg dry air
3	h, enthalpy	52.02002462463205	KJ/kg	52.540224870878376	kJ/(kg dry air)
4	s, entropy	0.1852597688313493	KJ/kgK	0.18711236651966281	kJ/(kg dry air K)
5	w, specific humidity, h...	0.01000000000000122	kg vap/kg dry air		
6	A, mass fraction of dry...	0.99009900990099	kg dry air/kg humid air		
7	xA, mole fraction of dr...	0.984176179326808	kmol dry air/kmol hu...		
8	relative humidity	0.44741544434487798			
9	Adiabatic saturation te...	18.519809229567613	degree C		
10	dew point temperature	13.840403083643821	degree C		
11	η Dynamic viscosity	4.481108595837139E-4	Pa.s		
12	k Thermal conductivity	0.027679038036385732	W/mK		
13	degree of saturation	0.44408995687375685			
14	Pa dry air partial pressure	98.4176179326808	kPa		
15	Pv water moisture parti...	1.582382067319199	kPa		
16	ρ density	1.1390376510069782	kg humid air/m³	1.1277600505019585	kg dry air/m³
17	Cp isochoric specific h...	1.0377745914519034	kJ/(kg humid air K)	1.0481523373664225	kJ/(kg dry air K)
18	Cv isovolumetric speci...	0.7261476481858238	kJ/(kg humid air K)	0.7334091246676822	kJ/(kg dry air K)
19	ha enthalpy of dry air ...	27.032583435393093	kJ/(kg humid air )	27.302909269747026	kJ/(kg dry air K)
20	hv enthalpy of vapor ...	2550.7641435484975	kJ/(kg water )		

Figure 4.3 humid\_air\_PR (Air as a single gas with pseudocritical properties) Graphic output for P=100 kPa and T=300 K

Properties of humid air Cubic Peng-Robinson EOS based. Air as a single gas					
array no	property name	value humid air mass fr b...	unit	value dry air mass fr base	unit
0	P, pressure	200.0	kPa		
1	T, temperature	26.850000000000023	deg C		
2	v, specific volume	0.43873354976100454	m³/kg humid air	0.44312088525861465	m³/kg dry air
3	h, enthalpy	52.035049806749115	KJ/kg	52.555400304816615	kJ/(kg dry air)
4	s, entropy	-0.015074846813934305	KJ/kgK	-0.015225595282073649	kJ/(kg dry air K)
5	w, specific humidity, h...	0.0100000000000000122	kg vap/kg dry air		
6	A, mass fraction of dry ...	0.99009900990099	kg dry air/kg humid air		
7	x <sub>A</sub> , mole fraction of dr...	0.9841761363142828	kmol dry air/kmol hum...		
8	relative humidity	0.8948333192390828			
9	Adyabatic saturation te...	26.839999999999918	degree C		
10	dew point temperature	24.973249865022353	degree C		
11	$\eta$ Dynamic viscosity	4.482400947149639E-4	Pa.s		
12	k Thermal conductivity	0.027719608086709372	W/mK		
13	degree of saturation	0.8942004153001419			
14	Pa dry air partial pressure	196.83522726285656	kPa		
15	Pv water moisture parti...	3.1647727371434398	kPa		
16	$\rho$ density	2.2792877374997635	kg humid air/m³	2.2567205321779835	kg dry air/m³
17	Cp isochoric specific heat	1.037771793661599	kJ/(kg humid air K)	1.0481495115982151	kJ/(kg dry air K)
18	Cv isovolumetric specif...	0.7262589710018084	kJ/(kg humid air K)	0.7335215607118265	kJ/(kg dry air K)
19	ha enthalpy of dry air K...	27.055010509378754	kJ/(kg humid air )	27.325560614472543	kJ/(kg dry air K)
20	hv enthalpy of vapor K...	2550.038979543754	kJ/(kg water )		

Figure 4.4 humid\_air\_PR (Air as a single gas with pseudocritical properties) Graphic output for P=200 kPa and T=300 K

Properties of humid air Perfect Gas EOS based					
array no	property name	value humid air mass fr b...	unit	value dry air mass fr base	unit
0	P, pressure	100.0	kPa		
1	T, temperature	-73.1499999999998	deg C		
2	v, specific volume	0.5856376284719803	m³/kg humid air	0.5914940047567002	m³/kg dry air
3	h, enthalpy	NaN	KJ/kg	NaN	kJ/(kg dry air)
4	s, entropy	Infinity	KJ/kgK	Infinity	kJ/(kg dry air K)
5	w, specific humidity, h...	0.0100000000000000122	kg vap/kg dry air		
6	A, mass fraction of dry ...	0.99009900990099	kg dry air/kg humid air		
7	x <sub>A</sub> , mole fraction of dr...	0.9841761363142828	kmol dry air/kmol hu...		
8	relative humidity	4745.217371539789			
9	Adyabatic saturation te...	-29.65477754876946	degree C		
10	dew point temperature	13.840444902461059	degree C		
11	$\eta$ Dynamic viscosity	2.0952363641611088E-5	Pa.s		
12	k Thermal conductivity	0.026829438769601054	W/mK		
13	degree of saturation	4773.772218751962			
14	Pa dry air partial pressure	98.41761363142828	kPa		
15	Pv water moisture parti...	1.5823863685717199	kPa		
16	$\rho$ density	1.7075405530364494	kg humid air/m³	1.6906342109271775	kg dry air/m³
17	Cp isochoric specific heat	NaN	kJ/(kg humid air K)	NaN	kJ/(kg dry air K)
18	Cv isovolumetric specif...	0.7263390357793207	kJ/(kg humid air K)	0.733602426137114	kJ/(kg dry air K)
19	ha enthalpy of dry air K...	-73.57492363560247	kJ/(kg humid air )	-74.3106728719585	kJ/(kg dry air K)
20	hv enthalpy of vapor K...	NaN	kJ/(kg water )		

Figure 4.5 humid\_air\_PG (Perfect Gas) Graphic output for P=100 kPa and T=300 K

Properties of humid air IAPWS formulation based					
array no	property name	value humid air mass fr ...	unit	value dry air mass fr base	unit
0	P, pressure	200.0	kPa		
1	T, temperature	26.850000000000023	deg C		
2	v, specific volume	0.43286549679599523	m³/kg humid air	0.43719415176439236	m³/kg dry air
3	h, enthalpy	51.75229077659324	KJ/kg	52.269813684411446	kJ/(kg dry air)
4	s, entropy	-0.011182786684057885	KJ/kgK	-0.011294614550909758	kJ/(kg dry air K)
5	w, specific humidity, h...	0.0100000000000000100007	kg vap/kg dry air		
6	A, mass fraction of dry ...	0.9900990099	kg dry air/kg humid air		
7	x <sub>A</sub> , mole fraction of dr...	0.9841761363127101	kmol dry air/kmol hu...		
8	relative humidity	0.894833193280208			
9	Adyabatic saturation te...	25.743134749487012	degree C		
10	dew point temperature	24.973247708116673	degree C		
11	$\eta$ Dynamic viscosity	4.480802598149723E-4	Pa.s		
12	k Thermal conductivity	0.02769973078997641	W/mK		
13	degree of saturation	0.8942004153895697			
14	Pa dry air partial press...	196.83522726254202	kPa		
15	Pv water moisture parti...	3.164772737457988	kPa		
16	$\rho$ density	2.310186437592851	kg humid air/m³	2.28731330454509	kg dry air/m³
17	Cp isochoric specific h...	1.0169360736338184	kJ/(kg humid air K)	1.0271054343711838	kJ/(kg dry air K)
18	Cv isovolumetric specif...	0.7253418356987908	kJ/(kg humid air K)	0.7325952540565114	kJ/(kg dry air K)
19	ha enthalpy of dry air ...	26.794473254711864	kJ/(kg humid air )	27.062417987286047	kJ/(kg dry air K)
20	hv enthalpy of vapor ...	2550.0389795436004	kJ/(kg water )		

Figure 4.3 humid\_air\_IAPWS Graphic output for P=100 kPa and T=300 K

Separately classes humid\_air\_IAPWS, and humid\_air\_PG is developed. The details of this classes will not be given here, but the results will be given for the same state, to compare the results.

Properties of humid air Cubic Peng-Robinson EOS based. Air as a single gas					
array no	property name	value humid air mass fr ba...	unit	value dry air mass fr base	unit
0	P, pressure	100.0	kPa		
1	T, temperature	26.850000000000023	deg C		
2	v, specific volume	0.8779341043871022	m <sup>3</sup> /kg humid air	0.8867134454309733	m <sup>3</sup> /kg dry air
3	h, enthalpy	52.02002462463205	KJ/kg	52.540224870878376	kJ/(kg dry air)
4	s, entropy	0.1852597688313493	KJ/kgK	0.18711236651966281	kJ/(kg dry air K)
5	w, specific humidity, hu...	0.010000000000000122	kg vap/kg dry air		
6	A, mass fraction of dry air	0.99009900990099	kg dry air/kg humid air		
7	x <sub>A</sub> , mole fraction of dry air	0.984176179326808	kmol dry air/kmol humi...		
8	relative humidity	0.4474154434487798			
9	Adyabatic saturation tem...	18.519809229567613	degree C		
10	dew point temperature	13.840403083643821	degree C		
11	$\eta$ Dynamic viscosity	4.481108595837139E-4	Pa.s		
12	k Thermal conductivity	0.027679038306385732	W/mK		
13	degree of saturation	0.44408995687375685			
14	Pa dry air partial pressure	98.4176179326808	kPa		
15	P <sub>v</sub> water moisture partial...	1.582382067319199	kPa		
16	$\rho$ density	1.1390376510069782	kg humid air/m <sup>3</sup>	1.1277600505019585	kg dry air/m <sup>3</sup>
17	C <sub>p</sub> isochoric specific heat	1.0377745914519034	kJ/(kg humid air K)	1.0481523373664225	kJ/(kg dry air K)
18	C <sub>v</sub> isovolumetric specific...	0.7261476481858238	kJ/(kg humid air K)	0.7334091246676822	kJ/(kg dry air K)
19	ha enthalpy of dry air KJ/...	27.032583435393093	kJ/(kg humid air )	27.302909269747026	kJ/(kg dry air K)
20	hv enthalpy of vapor KJ/...	2550.7641435484975	kJ/(kg water )		

Figure 4.5 humid\_air\_PR1 Graphic output for P=100 kPa and T=300 K

Properties of humid air Perfect Gas EOS based					
array no	property name	value humid air mass fr base	unit	value dry air mass fr base	unit
0	P, pressure	100.0	kPa		
1	T, temperature	26.850000000000023	deg C		
2	v, specific volume	0.8784017828286436	m <sup>3</sup> /kg humid air	0.8871858006569301	m <sup>3</sup> /kg dry air
3	h, enthalpy	51.99774272512549	KJ/kg	52.517720152376754	kJ/(kg dry air)
4	s, entropy	0.1851599720076865	KJ/kgK	0.1870115717277634	kJ/(kg dry air K)
5	w, specific humidity, humidity ratio	0.010000000000000122	kg vap/kg dry air		
6	A, mass fraction of dry air	0.99009900990099	kg dry air/kg humid air		
7	x <sub>A</sub> , mole fraction of dry air	0.9841761363142828	kmol dry air/kmol humid air		
8	relative humidity	0.4474166596195414			
9	Adyabatic saturation temperature	18.517703764710518	degree C		
10	dew point temperature	13.840444902461059	degree C		
11	$\eta$ Dynamic viscosity	4.4811104903645E-4	Pa.s		
12	k Thermal conductivity	0.02767903995508192	W/mK		
13	degree of saturation	0.44409115606801813			
14	Pa dry air partial pressure	98.41761363142828	kPa		
15	P <sub>v</sub> water moisture partial pressure	1.5823863685717199	kPa		
16	$\rho$ density	1.138431204886429	kg humid air/m <sup>3</sup>	1.1271596087984443	kg dry air/m <sup>3</sup>
17	C <sub>p</sub> isochoric specific heat	1.0377745914519034	kJ/(kg humid air K)	1.0481523373664225	kJ/(kg dry air K)
18	C <sub>v</sub> isovolumetric specific heat	0.7261483854545466	kJ/(kg humid air K)	0.7334098693090921	kJ/(kg dry air K)
19	ha enthalpy of dry air KJ/kg dry air	27.01007873556395	kJ/(kg humid air )	27.280179522919592	kJ/(kg dry air K)
20	hv enthalpy of vapor KJ/kg water	2550.7641416812485	kJ/(kg water )		

Figure 4.6 humid\_air\_PG Graphic output for P=100 kPa and T=300 K

## 5. CONCLUSIONS

In air conditioning processes in order to predict thermodynamic and thermophysical properties of humid air (a mixture of dry air and water vapor) usually perfect gas equation of state is utilised, But some processes such as drying of humid air in a compressed air tank, adding water to the compressor of gas turbine power plant to improve overall efficiency will require

better approaches. In thermophysical property predictions, most used approach is to assume such properties as only function of temperature. In reality properties such as viscosity and thermal conductivity heavily depends on pressure as well as temperature. Furthermore such properties are quite a nonlinear function of pressure so that a linear interpolation type of correction of properties will not be correct. In this study, a computer model of thermodynamic and thermophysical properties of humid air by using Peng-Robinson cubic equation of states is used. Two different approaches are utilised for dry air thermodynamic properties. In the first method, air is considered as a gas mixture and calculated from the gases in the mixture. In the second method air is considered as a single gas with pseudocritical properties and calculated by using these properties. Steam and dry air viscosity and thermal conductivity equations based on experimental studies by Kadoya et al[1] and Lemmon et al[2] is used. Water viscosity equations are taken from IAPWS Industrial Formulation 1997[4] and then these data combined by using Wilke equation[6]. Computer models for different set of real gas equation of State by IAPWS EOS and perfect gas formulations are also derived and results are compared. All computer codes developed in java programming language and program codes are given as free access to researchers at internet address [www.turhancoban.com](http://www.turhancoban.com). All equations for property of water-steam, dry air and humid air is given with details and coefficients for the researchers wish to develop their own version of computer codes in their desired programming language.

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